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Abstract
The aim of this paper is to present, in the scope of game theory, a new games representation: the code form, consisting of a table which contains the whole information, without any suppression or "adulteration "with regards to the game. After the theoretical description, its application is exemplified in solving the problem of selecting a candidate up to his/her acceptance trough game theory. The originality this paper proposes is how this problem will be approached: it will be treated as a single game which is made up of two parts, going as far as to state that the payoffs in the first part of the game will be the mediators of the second part of the game, instead of using the stochastic dynamic programming approach. The solution of the problem using stochastic dynamic programming is also presented to highlight empirically the parallelism between the two modes of resolution.

Keywords: *Game theory, two-part game, code form, stochastic dynamic programming.*

JEL Classification: C78
AMS Classification: 91A06

1 INTRODUCTION

The game here described represents a real life situation: candidate selection for a given job. It will be shown, because of the complexity of the relationships between different parties and the fact that each one of them is an essential element in the game's decision-taking, that the best candidate is not always the best option. On the other hand, through this game is also shown that knowing the fundamental elements that constitute a game - who are the players, what are the strategies for each player and the payoffs each player can receive - is not enough to know its solution, if one even exists.

This game is an example of game theory application to human resources management. On this subject see also (Andrade et al., 2012), (Ferreira et al., 2008, 2012, 2014, 2016), Ferreira (1991, 2014), (Filipe et al, 2012), (Matos and Ferreira, 2005) and (Matos et al., 2018).

2 GAME PRESENTATION

A presidential decree reduced the number of candidates to the vice-presidency to three people. Each of the three candidates are ranked on a scale from 1 (lowest) to 10 (highest).The presidential board attributed 10 points, 8 points and 5 points to the candidate classified in 1st place, 2nd place and 3rd place respectively. The probabilities of candidate i (i=1,2,3) accepting the j-th offer to run for the vice-presidency have been defined, considering that the first j-1 offers to the others have been declined, are denoted by p_{ij} and the respective values are in Table 1.

Table 1 Acceptance Probabilities

Player 1	$\boldsymbol{p_{11} = 0.5}$	$\boldsymbol{p_{12} = 0.2}$	$\boldsymbol{p_{13} = 0.0}$
Player 2	$\boldsymbol{p_{21} = 0.9}$	$\boldsymbol{p_{22} = 0.5}$	$\boldsymbol{p_{23} = 0.2}$
Player 3	$\boldsymbol{p_{31} = 1.0}$	$\boldsymbol{p_{32} = 0.8}$	$\boldsymbol{p_{33} = 0.4}$

The question is:
-What is the order in which the three potential candidates be offered the vice-presidential nomination if the presidential decree imposes the expected number of points maximization, supposing that no candidate is requested more than once and, each time a candidate rejects, another one is requested, until at least one has accepted or all have rejected.

3 GAME DETAILS

This game which is made up of two parts – a selection process and an acceptance process – attests that the payoffs in the first part of the game (potential candidates) will be the intermediaries of the second (decision elements).Thus:

Players:
Presidential Board:**4**
Potential Candidates:
Player classified in 1st place –**1**;
Player classified in 2nd place – **2**;
Player classified in 3rd place – **3**.

Strategies[1] **:**
Presidential Board:
The presidential board wants to establish the order in which the potential candidates will be invited to maximize the expected number of points. In this way the strategy for the presidential board will be the order in which the three potential candidates can be offered the vice-presidential nomination until at least one has accepted or all have rejected the offer - **P**.

Potential Candidates:
The strategy of each potential candidate is to accept the offer – **A** - or to reject the offer - **R**.

Payoffs:
Presidential Board:
The presidential board payoff is (a function of the attributed points in the daily preselection and of the potential candidates' probabilities of acceptance of the vice-presidency) the expected number of points of each possibility in the order of the proposal presented to the potential candidates. Thus, for instance: Player 1 rejects the offer, 3 rejects the offer, 2 accepts the offer,

[1] See (Bicchieri et al., 1999).

presidential board payoff is: $\frac{1}{2} \times \frac{1}{5} \times \frac{1}{5} \times 8 = 0.16$, where the probability of player 1 rejecting the offer is $\frac{1}{2}$, $\frac{1}{5}$ is the probability of player 3 rejecting the offer and $\frac{1}{5}$ is the probability of player 2 accepting the offer.

Potential Candidates:
For these players it is possible to define: if the player accepts the offer, he/she gets the "total prize", that is, he/she gets payoff 1. On the other hand, if he/she rejects the proposal, he/she does not get anything, so his/her payoff will be 0.

4 CODE FORME REPRESENTATION FOR THE GAME

To represent the game, code form game representation, see (Matos and Ferreira, 2002, 2002a, 2003, 2005), will be used. Code form is basically a table where the strategies that are available to any player are codified.

Definition 1:
A code form game consists of a finite table, with evident extension in the case of infinite moves and infinite players, where only some cells are filled. The cells will be filled in the same order the game is played. For that it is needed: $R = \{1,2, \dots, R\}$-a set of rounds; $J = \{1,2, \dots, J\}$-a set of moves; $C = \{1,2, \dots, J+3\}$-a set of columns; $L = \{1,2, \dots L\}$-a set of rows; $N = \{1,2, \dots, N\}$-a set of players; $E_N = \{e_1, e_2, \dots, e_N\}$-a set of strategies available to each player; $E = \{E_1 \times E_2 \times \dots . \times E_N\}$-a set of all such strategies profiles (space of strategies profiles); $\begin{matrix} RN: L \times \{1\} \to R \\ a_{i1} \to r \end{matrix}$, a function that indicates the round number; $\begin{matrix} PN: L \times \{2\} \to J \\ a_{i2} \to j \end{matrix}$, a function that indicates the move number; $\begin{matrix} JE: L \times C \to N \times E_N \\ a_{i,k} \to (N, E_N) \end{matrix}, c \neq 1,2, J+3$, a function that indicates who moves and what action is played; and $\begin{matrix} PJ: L \times \{J+3\} \to IR^N \\ a_{i,J+3} \to (u_1, u_2, \dots, u_N) \end{matrix}$, a function that gives the payoff of every player (for all the players) where $u_N: E \to IR$ is a von Neumann-Morgenstern utility function.■

Note:
-It can be denoted that when $RN(a_{i1}) = RN(a_{i-1,1})$, $PN(a_{i2}) = PN(a_{i-1,2})$ and $JE(a_{ic}) = JE(a_{i-1,c})$, the cells are not filled. The line is changed when the move changes. The column is changed when the player changes. ■

Code form game idea lies in the game estimated linear reading. The table is built containing the whole game information. In Table 2 is exemplified the code form game representation in the game example that is now considered. Reading from left to the right, the first column indicates the period number and the second column indicates the move[2] number. The following columns mention who moves when and in what circumstances and what action is played when somebody is called upon to move. Last column indicates the payoffs vector in accordance with the strategies chosen by the players. It is easy to check that the order in which the three potential candidates can be offered the vice-presidential nomination must be:
-To invite in the first place the candidate classified in 2nd place, 2; if he/she rejects the proposal, the candidate classified in third place, 3, should be invited and if he/she does not

[2] See (Benoit and Krishna, 1985) and Eberwein (2000).

accept, the candidate classified in first place, 1 should be invited. The expected number of points is 7.6.

Table 2 Code Form Game

1	1	(4,P)				
1	2		(1,A,0.5)			(5,1,0,0)
			(1,R,0.5)			
	3			(2,A,0.5)		(2,0,1,0)
				(2,R,0.5)		
	4				(3,A,0.4)	(0.5,0,0,1)
					(3,R,0.6)	(0,0,0,0)
	3			(3,A,0.8)		(2,0,0,1)
				(3,R,0.2)		
	4				(2,A,0.2)	(0.16,0,1,0)
					(2,R,0.8)	(0,0,0,0)
	2		**(2,A,0.9)**			(**7.2**,0,1,0)
			(2,R,0.1)			
	3			(1,A,0.2)		(0.2,1,0,0)
				(1,R,0.8)		
	4				(3,A,0.4)	(0.16,0,0,1)
					(3,R,0.6)	(0,0,0,0)
	3			**(3,A,0.8)**		(**0.4**,0,0,1)
				(3,R,0.2)		
	4				**(1,A,0)**	(**0**,1,0,0)
					(1,R,1)	(**0**,0,0,0)
	2		(3,A,1)			(5,0,0,1)
			(3,R,0)			
	3			(1,A,0.2)		(0,1,0,0)
				(1,R,0.8)		
	4				(2,A,0.2)	(0,0,1,0)
					(2,R,0.8)	(0,0,0,0)
	3			(2,A,0.5)		(0,0,1,0)
				(2,R,0.5)		
	4				(1,A,0)	(0,1,0,0)
					(1,R,1)	(0,0,0,0)

5 RESOLUTION THROUGH STOCHASTIC DYNAMIC PROGRAMMING

Now, a resolution using stochastic dynamic programming tools is presented. In this context this is a problem with three *stages*, representing the stage *j* the *j* th position in the invitation order. Define the *states* as the list of candidates still not invited. The stage 1 has only 1 state: $U_{11} = \{1,2,3\}$. The stage 2 has the three states: $U_{21} = \{1,2\}, U_{22} = \{1,3\}, U_{23} = \{2,3\}$ and the stage 3 has the three states: $U_{31} = \{1\}, U_{32} = \{2\}, U_{33} = \{3\}$.

Define also:

-$\boldsymbol{m_j(U_{jk})}$: Achieved points expected maximum number, beginning in stage j in state U_{jk}, supposing that there were no acceptations in the former stages,
-$\boldsymbol{d_j(U_{jk})}$: Candidate requested in stage *j* in order to get $m_j(U_{jk})$ and
-$\boldsymbol{V_i}$: Value in points of candidate *i*.

The recurrent formula, through which it is applied the so known backward induction process, is:

$$m_j(U_{jk}) = \max_{i \in U_{jk}} [V_i p_{ij} + (1 - p_{ij}) m_{j+1}(U_{jk} - \{i\})] \quad (1).$$

Follow its interpretation:
-If in state *j* the candidate *i* is requested and accepts, the value is V_i . Anyway if that candidate rejects, the best to go on is to depart from the state composed by the candidates that still were not requested. This formula is valid only for $j = 1,2,3$ if it is imposed that $m_4 = 0$.

So, going on,

Stage 3

$$m_3(U_{31}) = 10(0) = 0 \text{ with } \boldsymbol{d_3(U_{31}) = 1}$$
$$m_3(U_{32}) = 8(0.2) = 1.6 \text{ with } d_3(U_{32}) = 2$$
$$m_3(U_{33}) = 5(0.4) = 2.0 \text{ with } d_3(U_{33}) = 3$$

Stage 2

$$m_2(U_{21}) = \max\{10(0.2) + (1 - 0.2)m_3(U_{32}), 8(0.5) + (1 - 0.5)m_3(U_{31})\}$$
$$= \max\{10(0.2) + (1 - 0.2)(1.6), 8(0.5) + (1 - 0.5)(0)\} = 4 \text{ with } d_2(U_{21}) = 2$$
$$m_2(U_{22}) = \max\{10(0.2) + (1 - 0.2)m_3(U_{33}), 5(0.8) + (1 - 0.8)m_3(U_{31})\}$$
$$= \max\{10(0.2) + (1 - 0.2)(2.0), 5(0.8) + (1 - 0.8)(0)\} = 4 \text{ with } \boldsymbol{d_2(U_{22}) = 3}$$
$$m_2(U_{22}) = \max\{8(0.5) + (1 - 0.5)m_3(U_{33}), 5(0.8) + (1 - 0.8)m_3(U_{32})\}$$
$$= \max\{4 + (1 - 0.5)(2.0), 5(0.8) + (1 - 0.8)(1.6)\} = 5 \text{ with } d_2(U_{23}) = 2$$

Stage 1

$$m_1(U_{11}) = \max\{10(0.5) + (1 - 0.5)m_2(U_{23}), 8(0.9) + (1 - 0.9)m_2(U_{22}), 5(1) + (1 - 1)m_2(U_{21})\} = \max\{10(0.5) + (1 - 0.5)(5), 7.2 + 0.1(4), 5 + 0(4)\}$$
$$= \boldsymbol{7.6} \text{ with } \boldsymbol{d_1(U_{11}) = 2}$$

The the optimal policy is to request first the candidate 2. If he/she does not accept to request then the candidate 3 . And finally if this do not accept to request the candidate 1.

6 CONCLUSIONS

The analyzed game illustrates that the player who has decision power is not always the one who decides the game. In other words, the presidential board is the player who dominates the situation; therefore, they decide who the best candidate is. But, indeed, the ones who actually decide the game are the potential candidates when either they accept or they refuse the proposal. Really, the potential candidates are the ones who determine the game's outcome; starting from a situation of weakness they gain the control of the game. They are the result of the first part of the game and become the deciding elements of the second part of the game.

On the other hand, this game shows how much Game Theory is still a subject with a long way to go for games with more than two players, see Vega-Redondo (2003). No solution concept for these kinds of games is universally accepted. One reason for this is the situation described herein since one could not find the respective equilibrium using existing solution concepts, such as the Nash equilibrium, the Shapley value, and so on.

This in no way reduces the significance of Game Theory. In fact, besides motivating the theorists and considering the already developed results, it must not be forgotten Game Theory is one of the few theories that defines rational procedures in what were previously considered irrational situations and that its concepts and ideas have already provided very important and deep knowledge in the formulation of situations to real world conflicts.It is evident to conclude how much Game Theory is an asset to humanity, see for instance Gibbons (1992), (Kennan and Wilson, 1993), (Selvarasu et al, 2009) and Weibull (1995). Finally, note how the approach through Game Theory to the considered problem gives much more relevance to the human behaviour, that appears somehow hided in the approach through Stochastic Dynamic Programming.

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