Dynamic Optimization Method of Thermo-Mechanical Coupling Structure Based on ESLM

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Abstract: The equivalent static load method (ESLM) provides an effective solution for dynamic response optimization technology, and is used in simple structures such as trusses, beams, rods, and thin plates that bear a single mechanical load, or structures with lower grid requirements. Optimal design has been widely used, but there are few researches under the coupled effect of heat-mechanical load. Therefore, the research of dynamic optimization method of heat-mechanical coupling structure based on ESLM has certain significance and value. The thesis combines thermoelasticity with the equivalent static method theory, derives the equivalent static conversion mechanism of dynamic load under the coupling effect of thermal load and mechanical load, and uses cantilever beam as an example to verify the equivalent theory in thermophysical field and thermal-structure The same applies to coupled physics. Taking a typical transient thermal-mechanical coupling structure-a diesel engine piston as the research object, the stress and displacement results of 16 key positions are extracted through the transient dynamics analysis of the piston-thermal-mechanical coupling, and the equivalent conversion calculation is carried out. The comparative analysis of the load action results shows that the maximum equivalent stress error of the inspected area before and after the equivalent transformation is within 3%, and the maximum displacement error is within 2%; further analysis and comparison of the static optimization under the action of the piston thermomechanical load and the dynamic optimization based on ESLM The results show that the two optimized piston deformation, stress and temperature of the first ring groove are similar, but the ESLM-based dynamic optimization reduces the piston mass by 4.7% compared to the static optimization. The research results fully prove the superiority of the ESLM-based dynamic optimization method Sex.

Key words: Piston, Dynamic Optimization, Equivalent Static Load Method, Equivalent Thermal Load

1 Introduction

The equivalent static load method provides a new solution for structural dynamic optimization. However, the traditional ESLM basically studies the equivalent static transformation method under a single dynamic mechanical load. However, most of the mechanical Structures work under the joint action of multiple physical fields, so the optimization problem under the joint action of multiple loads has become a problem that plagues many scholars. For this reason, Dr. Jaehun Lee first proposed the equivalent static load method to solve the problem of large-scale structural dynamic optimization at the European Conference on Computational Fluid Dynamics. It should not be limited to only one physical field, but can be extended to fluid-structure coupled heat transfer and Acoustic structure in the coupled field[1]. Many scholars have conducted in-depth research on the thermal-structure coupling analysis and optimization and thermal equivalence of mechanical structures.

Gao Jianhui[2] combined the genetic algorithm with the "exhaustive search method" and used a two-step progressive strategy to conduct thermal-structural coupling analysis and optimization of the flame tube floating tile structure. The results confirmed that this optimization method is effective in tile thermal - Correctness in structural coupling optimization. Li Yulin[3] used the strong-weak coupling relationship to simplify the pneumatic-dynamic-thermal coupling problem, and established a pneumatic-dynamic-thermal multidisciplinary integrated analysis platform. For the time-consuming problems of different disciplines, an expanded adaptive response surface optimization strategy was adopted. The pneumatic-dynamic-thermal multidisciplinary design optimization was completed, which increased the flutter speed and reduced the quality of the lifting surface structure. Tang Xianlong[4] conducted analysis on the exhaust manifold considering only mechanical loads, only thermal loads, and thermal-structural coupling. The structure proved that thermal loads play a leading role in the damage of exhaust

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manifolds and should not be ignored in the analysis.

Finally, the exhaust manifold was optimized accordingly, and the stress and plastic deformation were reduced after optimization. Liu Yunfei[5] et al. directly calculated the unknowns required in the integral equation of the element equivalent node thermal load by using the overall coordinates of the element node, providing a new method for the equivalent thermal load of the node. Hu Xiangdong[6] put forward the idea of temperature field equivalence earlier, and used the temperature field of equivalent cross-section to equate the overall temperature field.

It can be seen from the findings of the above literature that although there are few equivalent static transformation studies under the combined action of thermal load and mechanical load, it has been verified in theoretical research and practical application. Therefore, for thermal load, mechanical load and thermal load -It is feasible to study the structural coupling load in the aspect of equivalent transformation. In view of the reason that most mechanical structures work together under dynamic thermal-structural coupling, it is urgent to study such problems to lay a solid theoretical foundation for dynamic response optimization.

2 Equivalent calculation theory of thermal-mechanical coupling load

2.1 Equivalent static load principle

The principle of equivalent static load: The equivalent static load of the structure in the linear static analysis can produce the system response field exactly the same as the nonlinear dynamic analysis of the structure at the corresponding time[7]. For the kinetic differential equation as equation (1):

$$M_D(b)\ddot{z}_D(t)+C_D(b)\dot{z}_D(t)+K_D(b)z_D(t)=f(t) \quad (1)$$

In the equation: M_D - Quality Matrix; C_D - Damping Matrix; K_D - Stiffness Matrix; $f(t)$ represents the external load vector at time *t*. According to the above principle, the equivalent static load expression is obtained:

$$f_{eq}(s)=K_L z_D(t) \tag{2}$$

In the equation: K_L -Stiffness matrix under equivalent static load; s represents the equivalent static load corresponding to each moment under dynamic load.

The specific equivalent principle is shown in Figure 1:

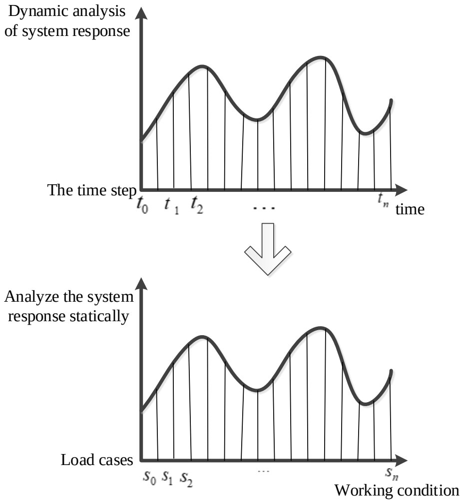

Fig 1 Equivalent Static Load Method Process

It can be seen from Figure1 that the calculation time step in the dynamic analysis is $n+1$ steps. When equivalent, each time step is regarded as a static working condition, and the system response of the S_i quivalent static load is required. The dynamic response is the same as the corresponding time step[7].

2.2 Thermal-Mechanical coupling equivalent mechanism

This article starts with the coupling problem of temperature field and strain field, and combines the linear thermal stress theory to deduce the equivalent static transformation of the heat conduction-force coupling problem.

Assuming that the elastic body is a slowly increasing quasi-static process under the action of thermal load and external load, the acceleration term and the influence of temperature change speed are ignored. Under this premise, the heat conduction equation and the equilibrium equation are independent, that is, the thermal conductivity equation of the temperature field is as follows Formula (3):

$$\lambda\nabla^2 T=\frac{\partial Q_1}{\partial\tau} \tag{3}$$

The heat conduction equation of the thermoelastic body, taking *T* and ε_{ij} as independent independent variables, starting from the Helmholtz free energy $F_1=U_1-T_s$, and finding the partial derivative at both ends according to equation (4)

$$\frac{\partial F_1}{\partial\varepsilon_{ij}}=\sigma_{ij} \qquad \frac{\partial F_1}{\partial T}=-s \tag{4}$$

Inferred:

$$\begin{aligned}\frac{\partial U_1}{\partial \varepsilon_{ij}} &= \frac{\partial F_1}{\partial \varepsilon_{ij}} + T\frac{\partial s}{\partial \varepsilon_{ij}} = \sigma_{ij} - T\frac{\partial}{\partial \varepsilon_{ij}}(\frac{\partial F_1}{\partial T}) \\ &= \sigma_{ij} - T\frac{\partial^2 F_1}{\partial T \partial \varepsilon_{ij}} = \sigma_{ij} - T\frac{\partial \sigma_{ij}}{\partial T}\end{aligned} \tag{5}$$

Substituting formula $\frac{\partial \sigma_{ij}}{\partial T} = -\beta\delta_{IJ}$ into formula (5), we get:

$$T\frac{\partial \sigma_{ij}}{\partial T} = -T\beta\delta_{ij} \tag{6}$$

Therefore:

$$\frac{\partial U_1}{\partial \varepsilon_{ij}} = \sigma_{ij} + T\beta\delta_{ij} \tag{7}$$

And for the isometric process, according to the formula $d\varepsilon_{ij} = 0$:

$$\frac{\partial U_1}{\partial T} = C_\varepsilon \tag{8}$$

Substituting formula (8) into it $dU_1 = \frac{\partial U_1}{\partial T}dT + \frac{\partial U_1}{\partial \varepsilon_{ij}}d\varepsilon_{ij}$, we can get:

$$dU_1 = C_\varepsilon dT + \sigma_{ij}d\varepsilon_{ij} + T\beta d\varepsilon_{ij}d\delta_{ij} \tag{9}$$

Substituting equation (9) and equation $ds = \frac{dU_1 - \sigma_{ij}d\varepsilon_{ij}}{T}$ into $\delta Q_1 \le Tds$, we get:

$$\begin{aligned}\delta Q_1 &= Tds = dU_1 - \sigma_{ij}d\varepsilon_{ij} \\ &= C_\varepsilon dT + T\beta d\varepsilon_{ij}d\varepsilon_{ij} \\ &= C_\varepsilon dT + T_0(\tfrac{T}{T0})\beta d\varepsilon_{ij}d\varepsilon_{ij} \\ &= C_\varepsilon dT + T_0(1+\tfrac{\Delta T}{T0})\beta d\varepsilon_{ij}d\varepsilon_{ij}\end{aligned} \tag{10}$$

When compared with is $\Delta T = T - T_0$ very small, $\Delta T / T_0$ can be approximately omitted, then:

$$\delta Q_1 \approx C_\varepsilon dT + T_0\beta d\varepsilon_{ij}d\varepsilon_{ij} \tag{11}$$

The above formula can be written as:

$$dQ\bullet d\tau = C_\varepsilon\frac{\partial T}{\partial \tau}d\tau + T_0\beta\frac{\partial \varepsilon_{ij}\delta_{ij}}{\partial \tau}d\tau \tag{12}$$

Where: dQ -The unit volume micro-element body exchanges heat with the outside per unit time, that is, the heat flow.

In addition, the balance of the three-dimensional model shows that:

$$dQ = \lambda(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}) = \lambda\nabla^2 T \tag{13}$$

Substituting formula (13) into (12), we get:

$$\lambda\nabla^2 T = C_\varepsilon\frac{\partial T}{\partial \tau} + T_0\beta\frac{\partial e}{\partial \tau} \tag{14}$$

Equation (14) is the heat conduction equation (modified Fourier heat conduction equation) after considering the deformation of the micro-element body of the thermoelastic object. Compared with equation (8), it is found that the second term on the right is an additional term caused by deformation work. It means that the elastic body is subjected to the external thermal load. One part of the structure temperature rise caused by the other part is transformed into the deformation of the elastic body. Compared with the traditional Fourier formula, the more part is called the coupling of the temperature field and the strain field. Item, which embodies the mutual coupling effect.

For the thermal-mechanical coupling equivalent static transformation problem, combining the linear thermal stress theory with the equivalent static transformation theory, we get:

$$\begin{aligned}f_{eq}^x(t) &= Kz_D^x(t) + Kz_T^x(t) \\ f_{eq}^y(t) &= Kz_D^y(t) + Kz_T^y(t) \\ f_{eq}^z(t) &= Kz_D^z(t) + Kz_T^z(t)\end{aligned} \tag{15}$$

In the equation: $f_{eq}^i(t)$ i=x,y,z -Equivalent static load in three directions; K -Stiffness matrix; $z_D^i(t)$ i=x,y,z -Displacement vectors in three directions of dynamic mechanical load; $z_T^i(t)$ i=x,y,z -Displacement components in three directions of thermal load.

2.3 Structural dynamic optimization design process based on equivalent static load method

It can be seen from the displacement-based equivalent static load formula (2). The equivalent static load can only be calculated after transient analysis of the structure. Explain that what is calculated by the equivalent static load is a known displacement field. From this perspective, the equivalent static load is meaningless, but the equivalent static load is ultimately used in structural optimization[8]. The specific steps of the optimized design are shown in Figure 2 below:

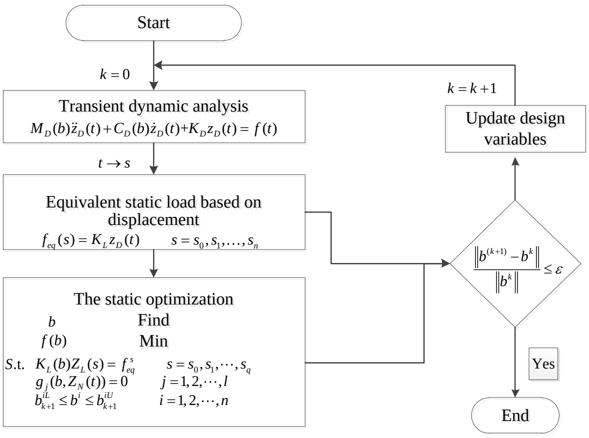

Fig 2 Equivalent Static Load Optimization Process

Figure 2 shows the optimization process based on the equivalent static load method. After the dynamic analysis of the structure, the stress, deformation or strain energy at key locations at key time points are selected, and the static load is extracted by the equivalent static load method for optimization.

2 Example verification

2.1 Simple entity structure verification

The following figure shows an aluminum alloy solid structure. Dynamic pressure is applied to the right end surface, and a temperature difference of 100°C and 20°C is applied to B and C, respectively, as shown in Figure3:

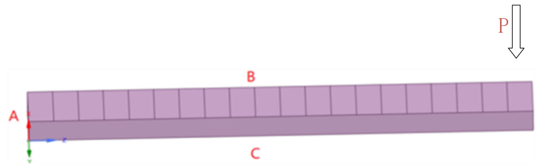

Fig 3 Aluminum Alloy Solid Structure

Select all nodes on the upper surface, perform equivalent static transformation in Isight, and select NLPQL algorithm for optimization. The optimization mathematical model is shown in equation (16), and the optimization iteration process is shown in Figure4:

$$\min\sum_{i=1}^{n}(u_i^s-u_i^d)^2 \qquad (16)$$
$$s.t.\ \ 0\le P_i\le 0.3MPa$$

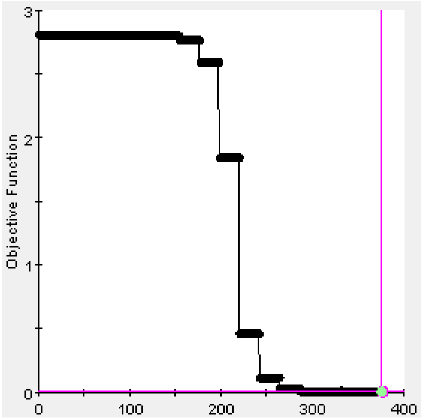

Fig 4 Optimize the iterative process

The results of the equivalent thermal load and mechanical load and the coupling analysis first and then the equivalent error results are shown in Figure 5:

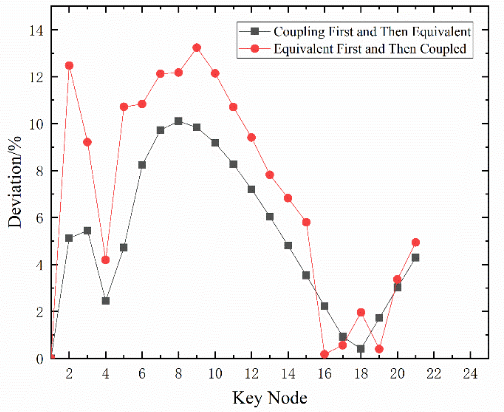

Fig 5 Error comparison of two different equivalent methods

It can be seen from Figure 5 that for the dynamic thermal-mechanical coupling problem, the accuracy of the equivalent after coupling is higher than the accuracy of the first equivalent and then the coupling, because for a thermal-mechanical coupling problem, considering the influence of the internal relationship between the thermal load and the mechanical load, However, the first equivalent and then the coupling is just a linear superposition of the same load. The mutual coupling effect is not considered, and a certain degree of accuracy is lost. However, for some large-scale ones, the mutual influence between different loads is not considered, and the coupling after the equivalence is also possible. Consider the method. But for the thermal-mechanical coupling equivalent in this paper, the method of coupling first and then equivalent is selected, and it is applied to the actual engineering structure-a diesel engine piston to prove the effectiveness of the method.

2.2 Practical application

2.2.1 Static optimization of a diesel engine piston

This paper selects the typical heat-receiving machine coupled load component-engine piston as the object for comparative analysis. In order to make the analysis more consistent with the actual operating conditions of the piston, this paper establishes a piston-connecting rod assembly model in Creo as shown in Figure 6:

Fig 6 Piston-Connecting Rod Assembly

The parameterized model of the piston cooling chamber established in the workbench is shown in Figure7:

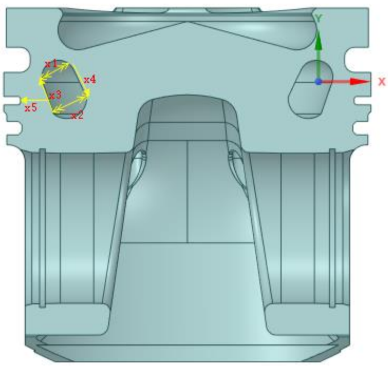

Fig 7 Parameterized Model of Piston Cooling Chamber

The grid size in the piston-connecting rod group defines the piston size as 1.5mm, and the rest is set to 3mm. The meshing control is tetrahedral meshing. The mesh of the contact surface is refined, and the defined size is 0.8mm. The number of division units is 276154 and the number of nodes is 463015. In the problem of piston-connecting rod assembly, because of its frictional contact and the friction coefficient is related to the lubrication, temperature and humidity between the contact surfaces, this article will compare the piston pin hole with the piston The friction coefficient between the inner surface of the small end hole of the pin and connecting rod and the cylindrical surface of the piston pin is set to 0.12; the contact algorithm is set to the penalty function method, because this algorithm is easier to converge on nonlinear problems; the first ring groove of the piston and The ring is set to bind contact, and treat it as a whole. The third type of boundary conditions is used in the analysis of the piston temperature field, the maximum pressure is selected as $p_{\max} = 14.5$ MPa, and the distribution is shown in Figure 8:

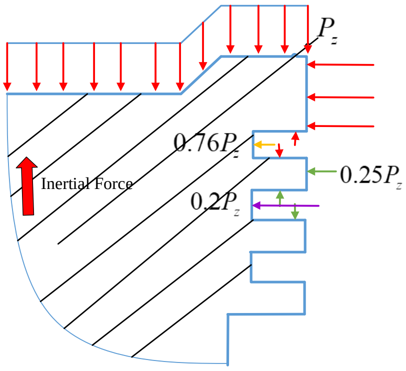

Fig 8 Piston pressure load boundary conditions

The static optimization of the piston takes the minimum mass of the piston as the optimization goal, the size of the piston cooling chamber as the design variable, the temperature and stress of the first ring groove as the constraint conditions, and the static optimization model is constructed as shown in equation (17):

$$\begin{aligned} &\min \ Mass \\ &s.t.\ \ \sigma_{\max} \le [\sigma] \\ &\qquad \mathrm{T}_1 \le 225℃ \\ &\qquad X_i \le X \le X_j \end{aligned} \tag{17}$$

In the equation: $Mass$ - Piston mass; $[\sigma]$ - The stress of the piston should not exceed the allowable stress value of the material; T_1 - The temperature of the first ring groove does not exceed the oil coking temperature of 225°C; X - Cooling cavity design variables; X_i, X_j respectively indicate the upper and lower limits of design variables.

In this paper, the response surface method is used to optimize the design, and the optimization algorithm selects the multi-island genetic algorithm. The optimized model is shown in Figure 9:

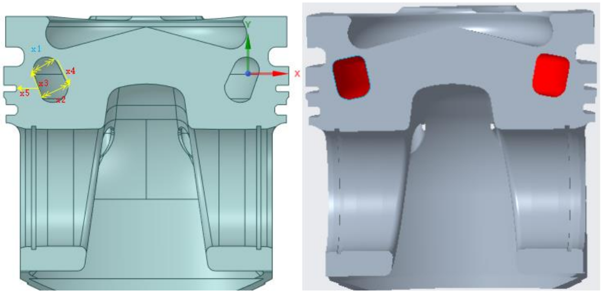

Fig 9 Comparison of cooling chamber before and after piston static optimization

The changes of mass and cooling cavity parameters before and after static optimization are shown below:

Table 1 Piston static optimization before and after comparison

Contrast variable	Before optimization	Later optimization
Mass/Kg	1.2605	1.2368
Deformation/mm	0.30833	0.32584
Maximum equivalent stress /MPa	173.87	181.83
X1/mm	10	12.7590
X2/mm	12	12.7591
X3/mm	10	13.7308
X4/mm	9	12.8964
X5/mm	7	6.7677

After the static optimization of the piston, the deformation and stress are increased by a small margin, and the mass is reduced by 1.88%. It can be seen that the optimization margin for the static optimization is relatively small.

2.2.2 *Based on ESLM piston dynamic optimization*

The equivalent static transformation method in this section draws on the energy equivalent static load method, which uses displacement and stress as equivalent static transformation indicators at the same time, and selects the displacement and stress of key nodes as shown in Figure10:

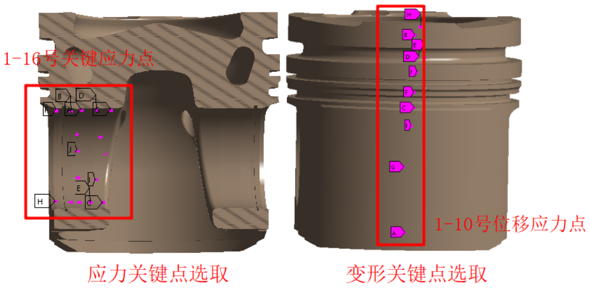

Fig 10 Key location point selection

The equivalent static transformation mathematical model is shown in formula (18):

$$\begin{aligned}
&\min \quad m=\sum_{i=1}^{16}(\sigma_i^S-\sigma_i^D)^2\\
&\min \quad n=\sum_{i=1}^{10}(x_i^S-x_i^D)^2\\
&\min \quad a=m+n\\
&s.t. \quad \mathrm{P}_1\le \mathrm{P}\le \mathrm{P}_2
\end{aligned}\qquad (18)$$

In the equation: σ_i^S -Equivalent stress for 16 key points; σ_i^D -16 key points transient analysis stress at the moment when the maximum equivalent stress appears; x_i^S -Equivalent displacement of 10 key points; x_i^D -Transient analysis of 10 key points Displacement at the moment when the maximum stress occurs; a is the objective function, which is the cumulative value of the sum of squares of stress differences at 16 key points and the sum of squares of displacement differences at 10 key points; design variables for the load value of each part of the piston, because the displacement and the maximum equivalent stress have a large difference in value, because The maximum equivalent stress is numerically larger than the displacement value. Therefore, the displacement and stress are set as two targets to solve the equivalent static load. The iterative convergence of the equivalent process is shown in Figure 11:

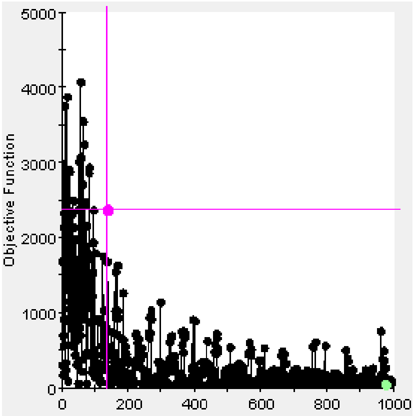

Figure 11 Iterative Convergence Graph

It can be seen from the scientific research in Figure 11 that the optimization process is iterated in 1002 steps in Isight. It can be seen that the target value is continuously reduced, and the range fluctuation is getting smaller and smaller with the optimization process, and the optimal solution is finally obtained. The equivalent load is shown in Table 2:

Table 2 The equivalent static load value of each area of the piston after optimization

Symbolic representation	Load loading area	Equivalent load value /MPa
1	Piston top surface	16.9980
2	Fire shore	16.7825
3	The upper surface of the first ring groove	16.7821
4	The inner surface of the first ring groove	8.6044
5	Lower surface of first ring groove	16.7821
6	The first roundabout	4.1731
7	Upper surface of the second ring groove	4.1733
8	The inner surface of the second ring groove	3.2382
9	Lower surface of second ring groove	4.1733

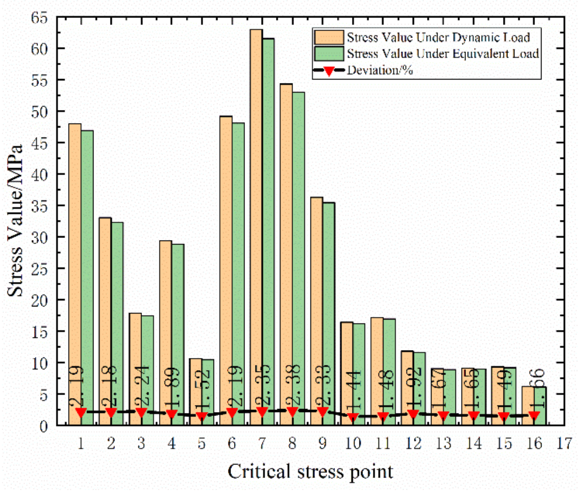

Fig 12 Key point displacement comparison chart

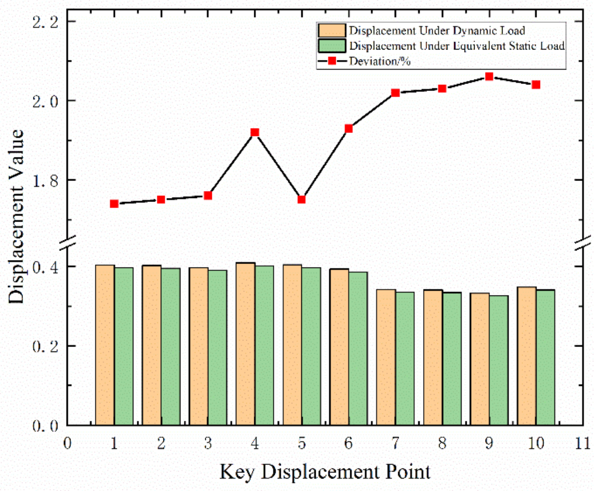

Fig 13 Comparison chart of maximum equivalent stress at key points

It can be seen from Figures 12 and 13, that the maximum error of the maximum equivalent stress value of the key positions before and after the equivalence is about 3%, which is within a reasonable acceptable range.

For the ESLM-based piston dynamic optimization mathematical model is the same as the statics, the optimized data based on the equivalent static load method is shown in Table 3:

Table 3 Comparison before and after dynamic optimization based on equivalent static load method

Contrast variable	Before optimization	Later optimization
Mass/Kg	1.2605	1.1788
Deformation/mm	0.30833	0.32584
Maximum equivalent stress /MPa	173.87	189.55
X1/mm	10	9.6487
X2/mm	12	13.709
X3/mm	10	13.138
X4/mm	9	13.769
X5/mm	7	5.673

It can be seen from Table 3 that the mass before and after optimization has been reduced by 6.5%, and the maximum equivalent stress and deformation have increased, but they are all within the allowable range of materials and structures, and compared with the static optimization results, the weight of the piston is improved. 4%, it can be seen that the dynamic optimization design based on the equivalent static load method is due to the static optimization design under purely extreme conditions.

6 Conclusions

This article explores the application of the dynamic response optimization method based on the equivalent static load method in the thermal-structure coupled field, combines the thermoelastic theory with the equivalent static method theory, and firstly derives the dynamic response under the coupled thermal load-mechanical load from a theoretical perspective. The load equivalent static conversion mechanism is verified by using the solid beam structure as a calculation example. By comparing the displacement values of the nodes before and after the equivalent, it is verified that the equivalent theory is also applicable under the thermal physical field and the thermal-structure coupled physical field. It is based on ESLM The dynamic optimization design of the thermal-mechanical coupling structure laid a theoretical foundation. And taking the typical transient heat-mechanical coupling structure-diesel engine piston as the research object, the stress and displacement results of 16 key positions are extracted through the transient dynamics analysis of the piston-heat-mechanical coupling, and the equivalent conversion calculation is carried out. The comparative analysis of the action results shows that the maximum equivalent stress error of the inspected area before and after the equivalent transformation is within

3%, and the maximum displacement error is within 2%. The equivalent transformation result is credible; further analysis and comparison of the static state under the action of the piston thermomechanical load Optimization and ESLM-based dynamic optimization. The results show that the two optimized piston deformation, stress and temperature of the first ring groove are similar, but the ESLM-based dynamic optimization reduces the piston mass by 4.7% compared to the static optimization. The research results fully prove the The superiority of ESLM dynamic optimization method.

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