Stabilization of Certain Classes of High-Order Systems Using a Robustness Fopid Controller

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Abstract— This paper presents a design procedure to tune the fractional order FOPID controller that stabilizes a certain class of higher order system. The procedure is based on assimilating an open loop response of higher order system as a first order system with delay. Using Ziegler-Nichols methods and these extensions, the parameters of the classic PID and fractional FOPID [20] correctors will be calculated to determine the closed-loop response. To verify the robustness of the fractional regulator, a random variation of the parameters of the selected third order process will be applied. The proposed approach has been verified by numerical simulation that confirms the effectiveness of the procedure.

Keywords— Fractional PID controller, delay system, stability, Zeigler-Nichols, robustness.

For decades, the PID regulator has been widely used in control engineering applications as it is the element that gives convincing results in many cases of industrial process control [1]. Practically the automation engineers use the method of Ziegler and Nichols which presented this method for the design and tuning of the PID corrector in 1942 [10]. The major drawback of this tuning method is the presence of poor performance, especially for high order systems (large overshoot and significant settling time). The fractional order PID regulator (FOPID) (or $PI^{\lambda}D^{\mu}$) was first proposed by IGOR Podlubny in 1999 [5-12]. Based on the fractional calculation, the fractional PIλDμ controller is characterized by five parameters: proportional gain (K_p), integration gain (K_I), derivative gain (K_d), integration order (λ) and derivative order (μ). Once the fractional $PI^{\lambda}D^{\mu}$ controller is compared to the classic PID controller, there are two parameters λ and μ making the former more efficient [13-14-16].

In some cases, real industrial systems are modeled as third order, fourth order or higher systems. Hence the need for an identification approach [2] that simplifies the stability analysis of certain high-order systems [23]. It is an approximation method mainly used by Ziegler-Nichols [3-8-9] which consists in assimilating the step response of these systems to that of a first order system with delay.

In general, a mathematical description is only an approximation of the actual physical system which deals with fixed nominal parameters. Usually, these parameters are not known exactly due to misidentification or imperfect measurement, aging of components and / or changes in environmental conditions [21]. In this paper we will try to adjust the parameters of the FOPID correctors in order to stabilize a high order system and prove its robustness [22].

II. FRACTIONAL CALCULUS AND FRACTIONAL ORDER DYNAMIC SYSTEMS

Fractional calculus is a generalization of integration and differentiation to non-integer order fundamental operator $t_0 D^m t$, where t_0 and t are the limits of the operation. The continuous integro-differential operator is defined as [5-6]:

$$ {}_{t_0}D_t^m = \begin{cases} \frac{d^m}{dt} & R(m) > 0 \\ 1 & R(m) = 0 \\ \int_{t_0}^{t} (d\tau)^{-m} & R(m) < 0 \end{cases} \quad (1)$$

$$ {}_{t_0}^{RL}I_t^m f(t) = \frac{1}{\Gamma(m)} \int_{t_0}^{t} (t-\tau)^{m-1} f(\tau) d\tau \quad (2)$$

with $t > t_0$ and $\Gamma(m)$ is Euler's gamma function.

The Riemann-Liouville derivative of order m of f with lower bound t_0 is defined by:

$$ {}_{t_0}^{RL}D_t^m f(t) = \frac{1}{\Gamma(n-m)} \frac{d^n}{dt^n} \int_{t_0}^{t} (t-\tau)^{n-m-1} f(\tau) d\tau \quad (3)$$

where $(n-1) < m < n$; n is an integer.

Finding approximations is very important for fractional-order transfer functions. In other words, fractional order transfer functions are replaced by whole order transfer functions when simulations need to be run where correctors need to be implemented. The Oustaloup simplified approximation [4] definition for fractional order differentiator is given in **Eq .4**.

$$ s^{\alpha} = K \prod_{m=1}^{M} \left[\frac{1+\frac{s}{\omega_{z,n}}}{1+\frac{s}{\omega_{p,n}}} \right], \propto > 0 \quad (4)$$

The approximation algorithm can be implemented to design a fractional order system as well as the fractional order controller [15-19]. **Fig .1** shows a block diagram of the fractional PID controller, which has the following structure

$$C_{FOPID}(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_I}{s^{\lambda}} + K_d s^{\mu} \qquad (5)$$

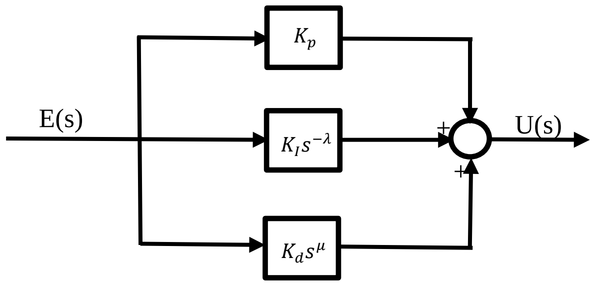

Fig .1: Block diagram of fractional PID controller.

A fractional PID controller becomes a PID controller if $\lambda = \mu = 1$ as shown in the **Fig. 2**.

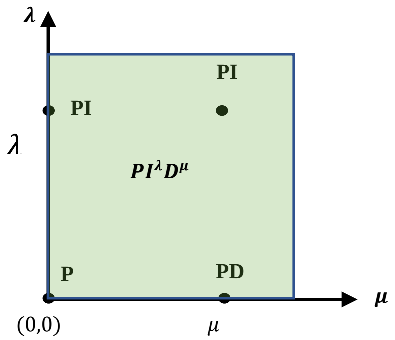

Fig .2: The fractional PID controller plane.

III- IDENTIFICATION

The methodology is very simple, for a certain class of high order linear systems which admit an aperiodic step response as shown at **Fig .3**, the system is characterized by a simplified model identified below [24]:

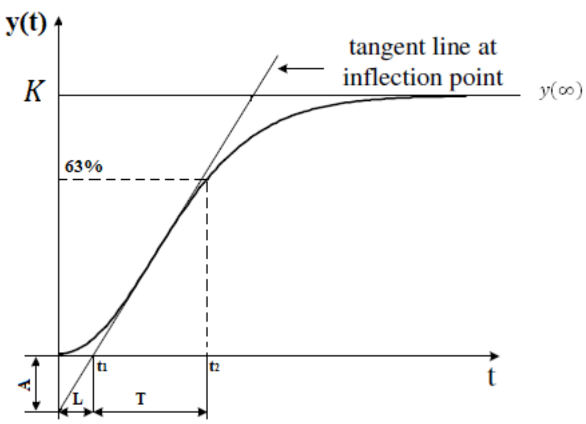

Fig .3: Open loop step response

We have highlighted the times t_1 and t_2. These times make it possible to define the following quantities [17]:

- The apparent delay:

$$L = t_1 \qquad (6)$$

- The apparent time constant:

$$T = t_2 - t_1 \qquad (7)$$

- The static gain:

$$K = \frac{y(\infty)}{E} \qquad (8)$$

With $y(\infty)$ is the asymptotic value and E is the amplitude applied at the input.

The model is then identified by the transfer function [7]:

$$G(s) = \frac{N}{a_n s^n + a_{n-1} s^{n-1} \dots \dots + a_1 s + a_0}$$

$$\backsimeq \frac{K}{1+Ts} e^{-Ls} \qquad (9)$$

IV- STABILIZATION OF FIRST-ORDER SYSTEMS WITH DELAY USING FOPID

The FOPID control loop is represented by the block diagram as shown in **Fig .4**, where G(s) and C(s) are the plant and the FOPID controller models, respectively. The FOPID receives the error signal E(s) and produces the control signal U(s) to control the output signal Y(s) and regulate the disturbance signal D(s), referring to the reference input R(s).

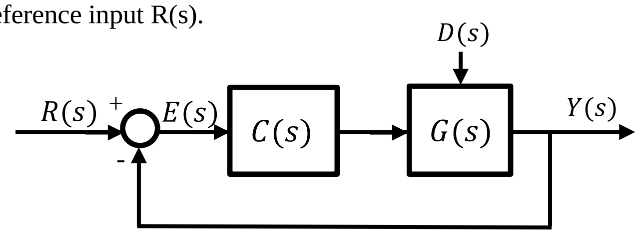

Fig .4: Closed loop system with fractional PID controller

The PID controller:

$$C_{PID}(s) = K_p + \frac{K_I}{s} + K_d s \qquad (10)$$

The fractional-order PID controller:

$$C_{FOPID}(s) = K_p + \frac{K_I}{s^{\lambda}} + K_d s^{\mu} \qquad (11)$$

IV-1- A FIRST METHOD OF ADJUSTMENT BASED ON THE OPEN LOOP RESPONSE

IV-1-a- USING PID CONTROLLER

The values of the PID parameters are given by Zeigler-Nichols

$$K_p = \frac{1,2}{A}\ ; \qquad K_I = \frac{0,6}{AL}; \qquad K_d = \frac{0,6L}{A} \qquad (12)$$

IV-1-b- USING FOPID CONTROLLER

In this method of adjustment, presented by Valerio and Costa in 2005 [18] and inspired by the Zeigler-Nichols.

$$K_p = P = -1.0574 + 24.5420L + 0.3554T$$
$$-46.7325L^2 - 0.0021T^2 - 0.3106TL \qquad (13)$$

	Parameters to use when $0.1 \le T \le 5$				
	P	I	λ	D	μ
1	−1.0574	0.6014	1.1857	0.8796	0.2778
L	24.5420	0.4025	−0.3464	−15.0846	−2.1522
T	0.3544	0.7921	−0.0492	−0.0771	0.0675
L^2	−46.7325	−0.4508	1.7377	28.0388	2.4387
T^2	−0.0021	0.0018	0.0006	−0.0000	−0.0013
LT	−0.3106	−1.2050	0.0380	1.6711	0.0021

Table .1: Parameter's adjustment of FOPID by the first method based on the open loop response

IV-2- A SECOND METHOD OF ADJUSTMENT BASED ON THE CLOSED LOOP RESPONSE

The critical point method of Zeigler-Nichols which gives a pumping phenomenon as shown in **Fig .5.**

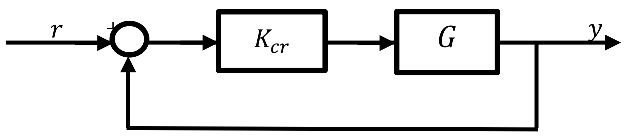

Fig .5: Control diagram

The period of such oscillations is the critical period Pcr, the gain which causes them is the critical gain K_{cr} as shown in **Fig .6**

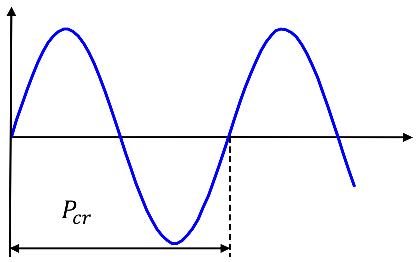

Fig .6. Pumping phenomenon

The values of the PID parameters are given as a function of Pcr and Kcr by the following formulas [11]:

$$K_p = 0.6 * K_{cr} \; ; \qquad K_I = 1.2 * K_{cr}/P_{cr} \; ;$$
$$K_d = 0.075 * P_{cr} * K_{cr} \qquad (14)$$

IV-2-b- USING FOPID CONTROLLER

It is an extension of the critical point method of Zeigler-Nichol. Also, Valerio and Costa were collecting the data in order to calculate the parameters K_p, K_I, λ, K_d and μ which vary regularly with K_{cr} and P_{cr}. The parameters of the polynomials involved are given in **table .2.** $Pcr \le 8$ and $Kcr * Pcr \le 640$

$$K_p = P = 0.4139 + 0.0145Kcr + 0.1584Pcr - \frac{0.4384}{Kcr} - \frac{0.0855}{Pcr} \qquad (15)$$

	P	I	λ	D	μ
1	0.4139	0.7067	1.3240	0.2293	0.8804
K_{cr}	0.0145	0.0101	−0.0081	0.0153	−0.0048
P_{cr}	−0.1584	−0.0049	−0.0163	0.0936	0.0061
$1/K_{cr}$	−0.4384	−0.2951	0.1393	−0.5293	0.0749
$1/P_{cr}$	−0.0855	−0.1001	0.0791	−0.0440	0.0810

Table .2: Parameter's adjustment of FOPID by the second method based on the closed loop response

V. SIMULATION RESULTS

In this section, the tuning method proposed is illustrated by a simulation example of high-order system given by:

$$G(s) = \frac{20}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} = \frac{20}{s^3 + 11s^2 + 32s + 25} \qquad (16)$$

V.1. IDENTIFICATION

The values of the parameters of the model are determined graphically following its step response in open loop by plotting the tangent at the point of inflection (**Fig .7**). We can then extract the variables K, T, L and A.

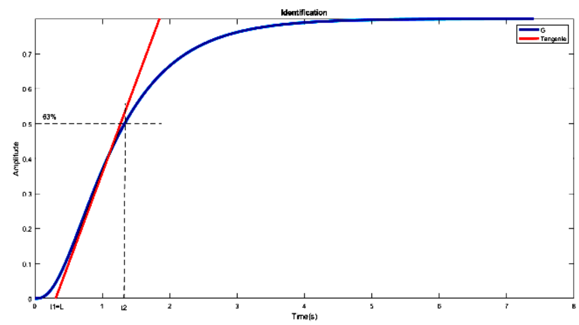

Fig .7: Open loop step response of G(s)

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K = 0.8 ; T = t2-t1=0.97 ; L = t1= 0.31
A = L/T=0.31
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$$\Rightarrow \quad G(s) \approxeq \frac{0.8}{1+0.97s} e^{-0.31s} \qquad (17)$$

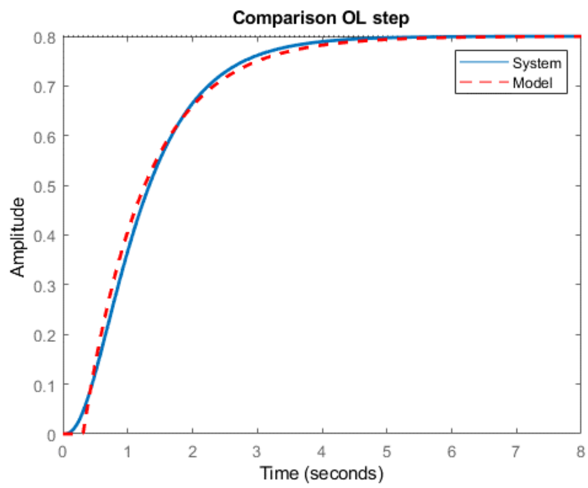

Fig .8: Comparison open loop step responses

In **Fig .8** the curve in blue (system) is the step response in open loop of our third order system while that in red (model) represents the response of the first order system with delay:

$$G(s) = \frac{20}{s^3 + 11s^2 + 32s + 25} \backsim \frac{0.8}{1 + 0.97s} e^{-0.31s} \quad (18)$$

V.2- STABILIZATION WITH A FIRST METHOD OF ADJUSTMENT BASED ON THE OPEN LOOP RESPONSE

We have

$$K = 0.8, \; T = 0.97\text{second and } L = 0.31. \quad (19)$$

Using **Eq. (12)** we can quickly calculate the three parameters of PID:

$$K_p = 3.75; \quad K_I = 6.05 \quad and \; K_d = 0.58 \quad (20)$$

$$C_{PID_1}(s) = 3.75 + \frac{6.05}{s} + 0.58s \quad (21)$$

Using **Table .1** we can quickly calculate the five parameters of FOPID:

$$K_p = P \;=\; 2.3080; \; K_I = I = 1.0905; \lambda = 1.2084;$$
$$K_d = D = -0.6747; \text{ and } \mu = -0.0901; \quad (22)$$

$$C_{FOPID_1}(s) = 2.308 + \frac{1.0905}{s^{1.2084}} \pm 0.6747s^{-0.0901}$$

(23)

The simulation gave the curve shown in **Fig .9:**

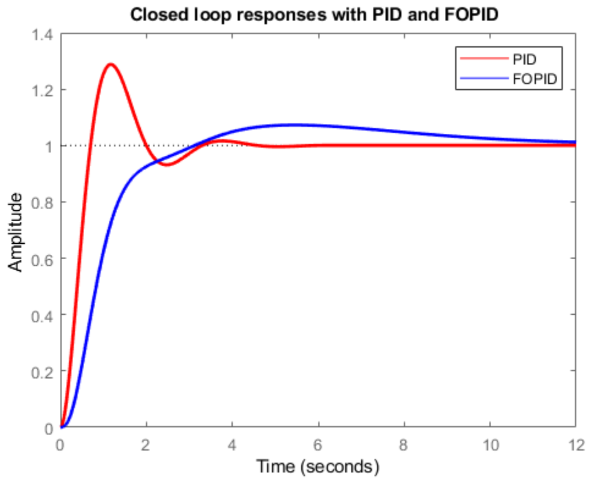

Fig .9: Closed loop responses with PID and FOPID using the first method of Zeigler-Nichols

Controller	Overshoot (%)	Risetime	Settling time
PID_1	28.7218	0.4819	2.7867
$FOPID_1$	7.2672	1.4252	7.7835

Table .3: Performance of various controller used

We propose to study the influence of a variation of the parameters of the coefficients of the denominator $[a_3 \; a_2 \; a_1 \; a_0]$ of ± 20% of $G(s)$ on the stability of the closed-loop system [25-26].

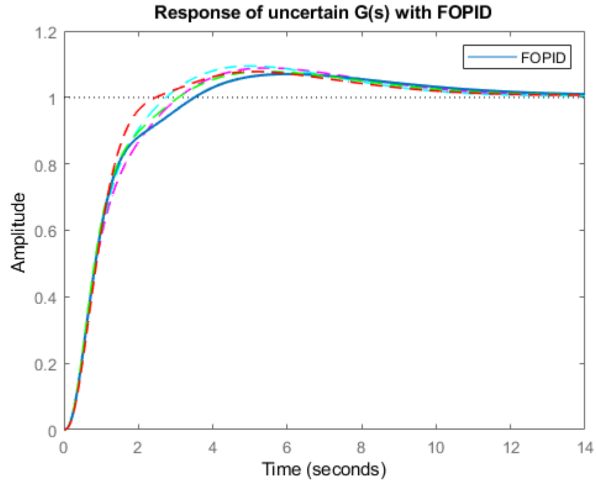

Fig .10 : Response in closed loop of $G(s)$ with an uncertainty of the parameters

Fig .11: Closed loop response of $G(s)$ with disturbance injection on its coefficients

A random choice of the coefficients of the denominator of $G(s)$ at a given instant ($t = 5s$) disturbs the output signal but it stabilizes quickly. This phenomenon is traced by the curve **Fig .11**

V.3- STABILIZATION WITH A SECOND METHOD OF ADJUSTMENT BASED ON THE CLOSED LOOP RESPONSE

At **Fig .12**, we present the phenomenon of pumping which gives us the critical period $P_{cr} = 1.11$ with a critical gain $K_{cr} = 16.36$.

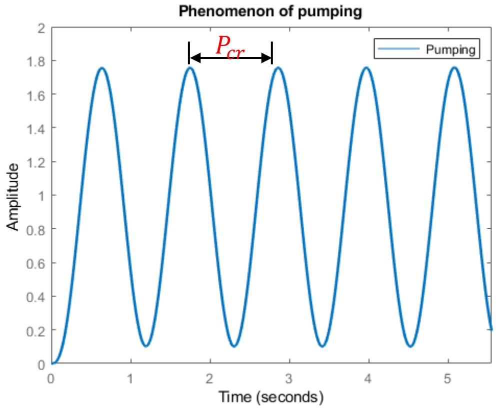

Fig .12: Pumping ' s phenomenon

We find the three parameters of PID from the **Eq. (14)**
$K_p = 9.81$; $K_I = 17.68$ and $K_d = 1.36$.

$$C_{PID_2}(s) = 9.81 + \frac{17.68}{s} + 1.36s \qquad (24)$$

The five parameters of the FOPID corrector were calculated using the **table .2**

$$Kp = 0.7232;\ K_I = I = 0.7583\ ;\ \lambda = 1.2531;$$

$$K_d = D = 0.5116 \quad and \quad \mu = 0.8862 \qquad (25)$$

$$C_{FOPID_2}(s) = 0.7232 + \frac{0.7583}{s^{1.2531}} + -0.5116s^{-0.8862} \qquad (26)$$

So, we can then we can plot the closed loop response of g(s) with the two regulators PID and FOPID:

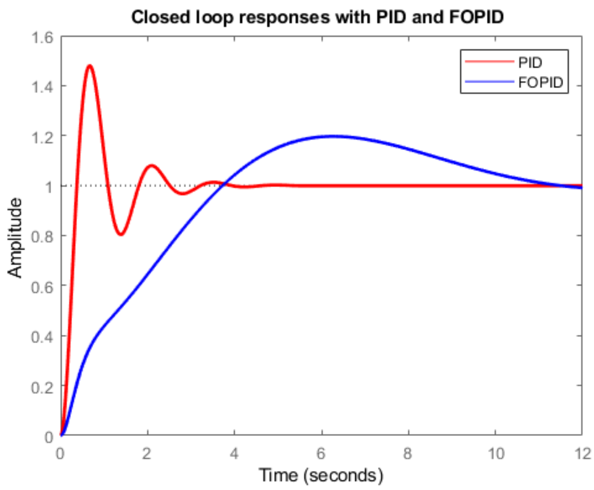

Fig .13: Closed loop responses with PID and FOPID using the second method of Zeigler-Nichols

Controller	Overshoot (%)	Risetime	Settling time
PID_2	47.9387	0.2529	2.3058
$FOPID_2$	19.6825	2.9559	9.9384

Table .4: Performance of various controller used

At **Fig .14**, colors show the performance change gaps of systems with ±20% coefficients uncertainties. Stability with the proposed Robust FOPID controller is guaranteed which means that the system is fairly robust to gain variations.

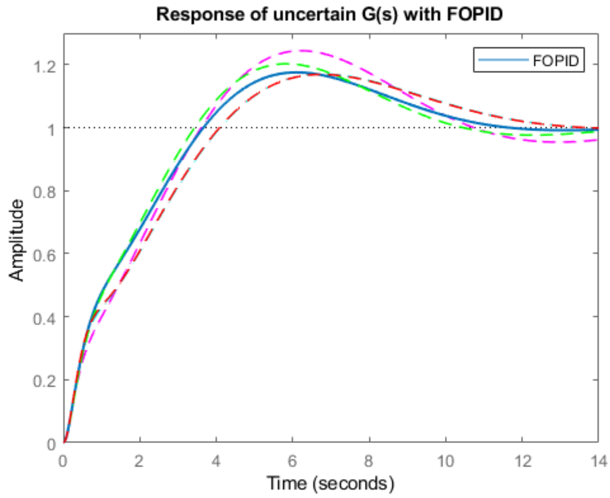

Fig .14: Response in closed loop of $G(s)$ with an uncertainty of the parameters

Another vital concern for control system is their disturbance rejection ability. Therefore, an external disturbance is added to $G(s)$ at t =23s. **Fig 15** show the disturbance rejection of the systems. It is illustrated that systems controlled by Robust FOPID controller return back to the set-point value quickly after the appearance of external disturbance.

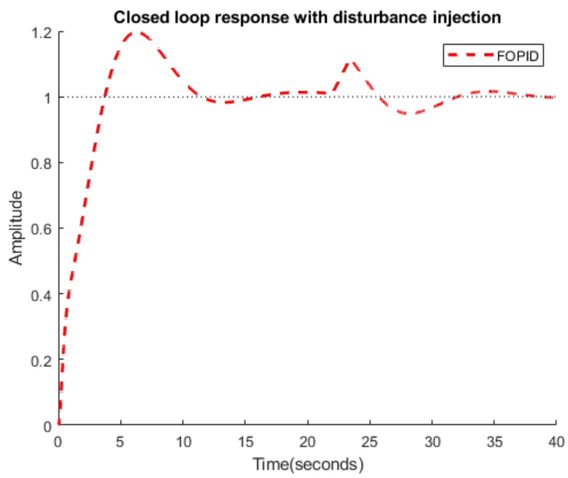

Fig .15: Closed loop response of $G(s)$ with disturbance injection on its coefficients

VI. CONCLUSION

In this note, a useful stabilization method was used to obtain the parameters stabilizing fractional order PID controller for a high order system. The basis of this approach is to determine a first order simplified model with delay and then use the parameters of this model to calculate both classical PID and fractional FOPID correctors. A comparative study was carried out to show the effectiveness of the new fractional corrector from a performance point of view with the two Zeigler-Nichols methods. The robustness of the corrector has also been tested.

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