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EIGENVALUE ANALYSYS OF GRAPH LAPLACIAN REPRESENTING HELIX DEVIATION NETWORK

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1. INTRODUCTION

A novel approach for assessing the geometry of gears generated through hobbing, dubbed the shape deviation network, was introduced by Iba et al [1][2]. This methodology facilitates the visualization of phase differences among deviations from a holistic perspective. Building upon this, Kono et al. developed a network delineating tooth profile and helix deviations, employing artificial intelligence for image recognition to estimate hobbing-induced errors [3][4]. However, investigations into the distinctive attributes of hobbed gear's shape deviation network are notably lacking. In this investigation, we first construct a network utilizing tooth helix deviation curves of hobbed gears, followed by eigenvalue analysis of the graph Laplacian to quantitatively assess the intrinsic characteristics of the tooth helix deviation network.

2. HOBBING SIMULATION AND GEAR SPECIFICATIONS

In this manuscript, the shape deviation curves of a gear were derived employing the hobbing simulation software engineered by Kono (3). Table 1 shows the specifications of the gears to be hobbed. Additionally, Figure 1 illustrates the tooth helix deviation curve generated by the simulation.

Table 1. Gear Specification.

Gear data	
Normal Module [mm]	2
Number of Teeth [-]	30
Normal Pressure Angle [deg.]	20
Helix angle [deg.]	30
Face width [mm]	15

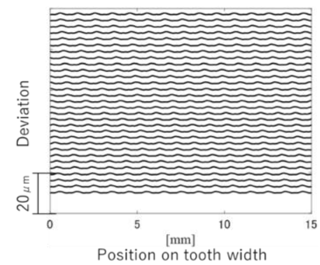

Fig. 1 Helix deviation curves

3. CORRELATION COEFFICIENT

A network is constructed from the shape deviation curves of the hobbed gear. In this section, we first explain the correlation coefficients that act as weights among nodes in the tooth profile deviation network. In this manuscript, the correlation coefficients are employed as quantitative metrics for phase differences in the tooth helix deviations. First, let $f_{j\beta}(x)$ and $f_{k\beta}(x)$ be two tooth helix deviation curves, where j and k are positive natural numbers from tooth number 1 to z and x is a variable in the tooth width direction. The inner product between the two tooth helix deviation curves is defined as in Equation (1). In this equation, L means the range of integration.

$$\langle f_{j\beta}(x), f_{k\beta}(x)\rangle = \frac{1}{L}\int_0^L f_{j\beta}(x) f_{k\beta}(x) dx \quad (1)$$

Next, we define the norm of the jth tooth deviation curve $f_{j\beta}(x)$ in equation (2).

$$\left\| f_{j\beta}(x) \right\| = \sqrt{\frac{1}{L}\int_0^L \left\{ f_{j\beta}(x) \right\}^2 dx} \quad (2)$$

Using these inner products and the norm of the deviation curve, we define the correlation coefficient between the two tooth helix deviation curves in equation (3).

$$r_{j,k} = \frac{\langle f_j(x), f_k(x)\rangle}{\left\| f_j(x) \right\| \left\| f_k(x) \right\|} \quad (3)$$

Here, the defined correlation coefficient takes the range $-1 \leq r_{j,k} \leq 1$. If this coefficient is close to 1, it means that the phase difference between the two deviation curves is small, and vice versa. Furthermore, in order to consider the eigenvalue analysis of the graph Laplacian matrix as a mechanical vibration system composed of springs and masses, the defined correlation coefficient is normalized to vary from 0 to 1 according to the following equation.

$$R_{j,k} = \frac{r_{j,k}+1}{2} \tag{4}$$

4. EIGENVALUE ANALYSIS OF GRAPH LAPLACIAN OF HELIX DEVIATION NETWORK

We establish the tooth helix deviation network by defining the coefficients as links among the nodes according to equation (4). The resultant network can be characterized as an adjacency matrix. The adjacency matrix A of the network with a total of N nodes becomes an $N \times N$ square matrix, where typically, the elements $A_{m,m}$ of the adjacency matrix, where the link (m, n) has a weight $w_{m,n}$, are defined as follows:

$$A_{m,n} = \begin{cases} w_{m,n} & (\text{If } v_m \text{ and } v_n \text{ are connected}) \\ & 0 \ (\text{In other case}) \end{cases} \tag{5}$$

The sum of the weighted links connecting each node is called the degree, and the matrix whose diagonal components are the degree is called the degree matrix D. The diagonal component D_{ii} of the degree matrix is represented by the following equation (6).

$$D_{ii} = \sum_{j=1}^{n} A_{ij} \tag{6}$$

Using equations (5) and (6), the graph Laplacian matrix of the network is defined by the following equation

$$L = D - A \tag{7}$$

Attempts to quantitatively evaluate the network created by performing an eigenvalue analysis of the graph Laplacian matrix.

5. RESULT AND DISCUSSION

Constructing a tooth helix deviation network from the correlation coefficients between the tooth helix deviations, we present the results of eigenvalue analysis conducted on the obtained graph Laplacian. In Figure 2 left, the vertical axis represents the eigenvalues, and the horizontal axis represents the degrees of the eigenvalues. Due to the properties of the graph Laplacian, the smallest eigenvalue is 0. Furthermore, the eigenvalues of the second and third are of the same magnitude, as are the eigenvalues of the fourth and fifth, and the sixth and seventh.

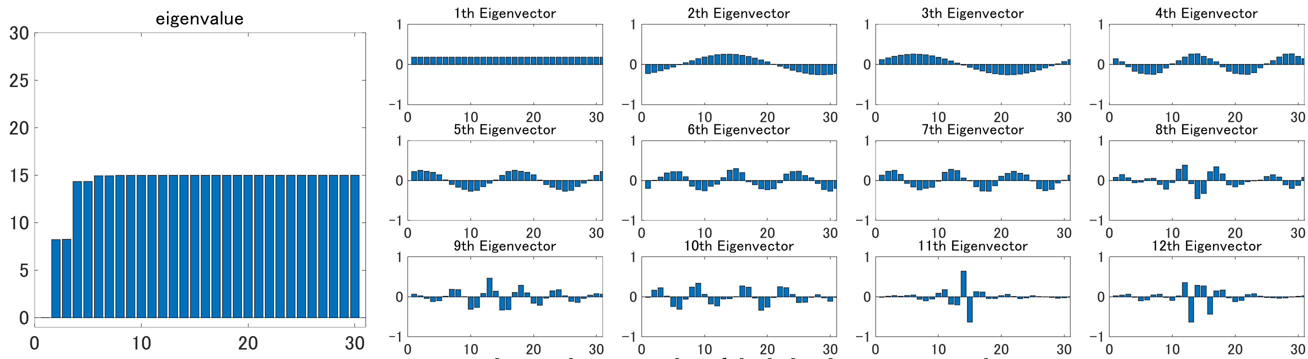

Figure 2. Eigenvalues and eigen modes of the helix deviation network

By normalizing the correlation coefficients, and since all coefficients take positive values, eigenvalue analysis of the graph Laplacian can be conceptualized as being replaced by a mechanical vibrational system composed of springs and masses. Here, the nodes of the network represent mass points, and the correlation coefficients can be regarded as springs connecting masses. Thus, eigenvalues can be interpreted as natural frequencies, and eigenvectors as corresponding vibration modes. Due to the properties of the graph Laplacian, there exists a zero-eigenvalue corresponding to a rigid mode with uniform amplitude. Subsequent eigenvalues appear in pairs, and their corresponding eigenmodes exhibit shapes with a 90-degree phase difference, resembling sine and cosine waves. By expressing it in this way, it has become possible to interpret the inherent characteristics of the tooth profile deviation network of hobbed gears more easily

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