Robust Angular Velocity Control of a VTOL-UAV for Aggressive Maneuvers Flight

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Abstract This work presents the design of an autopilot for acrobatic maneuvers such as multiple flips, invert itself completely and backflips. This flight mode is possible by controlling the angular velocity of the UAV instead of the attitude. A robust angular velocity control is approached that allows to follow aggressive references at high speeds to generate acrobatic flights. The algorithm uses the active disturbance rejection control ADRC to estimate and eliminate effects that are generated in this type of flights, such as: unpredictable dynamics, external disturbances and ground effect. The control algorithm is parameterized by quaternion avoiding the gimbal lock singularities, in addition the maximum torques of the rotors are taken into account by means of a saturation function.

1 Introduction

In recent years the development of unmanned aerial vehicles for vertical take-off and landing (VTOL-UAV) have received notable growth. The quadcopter is the standard platform for the development of control systems, trajectory generation, cooperative control, aerial manipulation and aggressive maneuvers. Undoubtedly, its high maneuverability, simple design and reduced size are factors that favor its use in applications such as: search and rescue, precision agriculture, aerial photography, disaster relief, among others. However, designing control algorithms for stabilization of their states is not trivial, since the quadcopter is a nonlinear system, underactuated, perceptible subject to external disturbances and parametric uncertainties. Various control approaches have been published in the scientific literature to perform aggressive maneuvers. For example in [1], the problem of multiple flips is addressed using linear control of angular velocity and iterative adaptation for parameter correction.

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In [2] the dynamics of the system is simplified by hybrid systems, achievable sets are used to perform an autonomous backflip. A linear control that stabilizes the angular velocity and a trajectory generation for the generation of multiple flips are proposed in [3]. Another very different approach to controlling angular velocity is trajectory generation by waypoints specifications. The translation of the UAV is minimized by a quadratic program [4], [5], [8]. Other approaches like learning automata algorithm [6] and reinforcement learning [7]. This research addresses the problem of designing an autopilot for acrobatic maneuvers such as multiple turns, inversion of the quadcopter completely and inverted somersaults. This flight mode is possible by controlling the angular velocity of the UAV instead of the euler angles. Performing high speed maneuvers will generate unwanted effects such as high oscillation, transients, turbulence and ground effects which are difficult to model. For this reason, a control with active disturbance rejection is proposed ADRC, which by means of an extended state observer (ESO) estimates the total disturbance online. Once the total disturbance has been estimated, a bounded control based on quaternions is used together with the action of the observer to close the control loop. The algorithm is relatively simple to implement in embedded systems with limited processing power.

2 Mathematical Model of the Quadrotor.

The quadcopter consists of a rigid transverse frame that contains four rotors at its ends. To derive the equations that describe the attitude and position of the VTOL-UAV, it is necessary to consider two orthogonal coordinate frames = $\{\mathbf{e}_1^b, \mathbf{e}_2^b, \mathbf{e}_3^b\}$, located at the center of mass of the quadcopter and the inertial coordinate frame, = $\left\{\mathbf{e}_1^f, \mathbf{e}_2^f, \mathbf{e}_3^f\right\}$, located at some point in the Earth's surface, also is chosen as the north-east-down (NED) frame. Like any rigid body, the attitude in space is described by the rotation between the frames and . The parametrization by quaternions are used to describe the attitude allowing nonlinear large angles, aerobatic and high speed aggressive maneuver. Introducing the quaternion $q \in \mathbb{S}^3$ as:

$$q := \begin{pmatrix} \cos\frac{\beta}{2} \\ e\sin\frac{\beta}{2} \end{pmatrix} := \begin{pmatrix} q_0 \\ q_v \end{pmatrix} \in \mathbb{S}^3 \tag{1}$$

where e is a unit vector and β is the rotation about of this axis, furthermore $q_v = \left(q_1\ q_2\ q_3\right)^T \in \mathbb{R}^3$ and $q_0 \in$ are known as the vector and scalar parts of the quaternion respectively. Satisfy the following equation:

$$q^T q + q_0^2 = 1 \tag{2}$$

The unit quaternion q represents the rotation of the frame to through the map $R_b^f : \mathbb{S}^3 \to SO(3)$ defined as:

$$R_b^f(q) := I_3 + 2q_0[q_v^\times] + 2[q_v^\times]^2 \tag{3}$$

$[r^\times]$ is the well known skew-symmetric matrix associated to vector r.
Then, the mathematical model that describes the kinematic equation of rotation of the quadcopter is given by:

$$\dot{q} = \frac{1}{2}\begin{pmatrix} -q_v^T \\ I_3 q_0 + [q_v^\times] \end{pmatrix}\omega := \frac{1}{2}\Xi(q)\omega \tag{4}$$

where $\omega = \left(\omega_1\ \omega_2\ \omega_3\right)^T \in^3$ be the angular velocity vector of the body coordinate frame . The attitude error is used to quantify the mismatch between two attitudes. If q defines the current attitude quaternion and q^d the desired quaternion, *i.e.*, the desired orientation, then the quaternion that represents the attitude error between the current orientation and the desired one is given by:

$$\tilde{q} = (q^d)^{-1} \odot q = \left(\tilde{q}_0\ \tilde{q}_v^T\right)^T \tag{5}$$

where q^{-1} is the complementary rotation of the quaternion q, which is given by $q^{-1} = \left(q_0\ -q_v^T\right)^T$ and $\odot$ denotes the quaternion multiplication [9]. When the current quaternion q reaches the desired one q^d, the quaternion error becomes $\tilde{q} = \left(\pm 1\ 0^T\right)^T$, *i.e.*, there exist two equilibria which have to be considered in the stability analysis [10]. The dynamic model is infer analyzing the forces and moments that influence on the quadcopter, the attitude dynamics is mathematically described as:

$$\dot{\omega} = -[\omega^\times]J\omega + G_a + \Gamma + \xi(t) \tag{6}$$

where $J \in^{3\times 3}$ denotes the inertia matrix, constant and symmetric, $G_a = \left(G_{a,\phi}\ G_{a,\theta}\ G_{a,\psi}\right)^T \in^3$ mean the gyropic term caused by the combination where the rotation of the aircraft and the four rotors, $\xi(t)$ is a time-dependent external additive disturbance possibly due to the effect produced by the coupled system, wind, turbulence or varying mass of the system, the vector $\tau = \left(\tau_x\ \tau_y\ \tau_z\right)^T \in^3$ represents the moments generated by the difference between the angular velocity of the rotors, as a consequence, it represents the control signal. The main objective of the present paper is the design of a robust control able to follow a varying angular velocity $\omega_d = \left(\omega_{1,d}\ \omega_{2,d}\ \omega_{3,d}\right)^T \in^3$ in time. Also, the UAV-VTOL is subject to external disturbances, parametric uncertainties, uncertain and unpredictable dynamics. The above described can be formalized mathematically as:

$$q(t) \to q_d(t),\ \omega(t) \to \omega_d(t), \zeta(t) \to \epsilon,\ \text{as } t \to \infty \tag{7}$$

where $\omega_d(t)$ is the time-dependent desired velocity, usually is provided by a radio-control, $q_d(t)$ is calculated from the kinematic equation (4) substituting ω by ω_d, $\zeta(t) = \left(\zeta_1\ \zeta_2\ \zeta_3\right)^T \in^3$ is the total disturbance which will be defined in the next section and ϵ it is a very small scalar number. In this work, an Active Disturbance Rejection Control (ADRC) is designed to tracking the desired angular velocity $\omega_d(t)$ of the VTOL-UAV. An Extended State Observer (ESO) complements the control algorithm. The saturation function is used to limit a region of operation to the angular velocity of

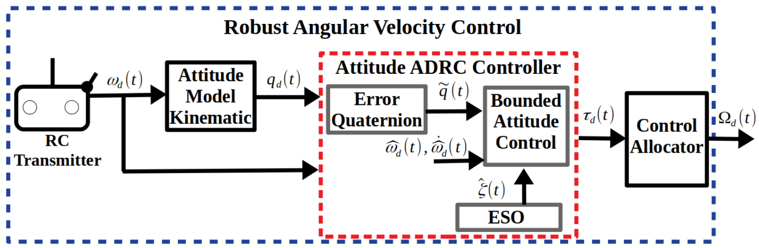

Fig. 1: proposed control algorithm, (Attitude Model Kinematic, Attitude ADCR controller and ESO).

the quadrotor. The controller is parameterized by quaternions, avoiding singularities and ensuring aggressive maneuvers. Also, the control strategy takes into account the maximum torques allowed by the rotors. This can be formulated as:

$$\Gamma_j \in [-\bar{\Gamma}_j, \bar{\Gamma}_j], \quad j \in \{1, 2, 3\}$$

where $\bar{\Gamma}_j$ represents the bounds of the j^{th} control torque component.

3 Robust Angular Velocity Control

The structure of the proposed control algorithm is divided into three blocks, *Attitude Model Kinematic*, *ESO* and *Attitude ADRC Controller* the Fig.1. The desired angular velocity $\omega_d(t)$ is provided by an RC transmitter, then the desired quaternion $q_d(t)$ is obtained using the kinematic equation (4), concurrently an Extended State Observer ESO estimate the angular acceleration $\dot{\omega}_d(t)$ and angular velocity $\omega_d(t)$. The $q_d(t)$, $\dot{\hat{\omega}}_d(t)$ and $\hat{\omega}_d(t)$ variables are used to calculate the control torques $\tau_d(t)$ by an internal control block that stabilizes the orientation trajectory tracking, in this block the ADRC is used in conjunction with a bounded quaternion-based feedback. Finally an allocator controller calculates the speeds of the rotors $\Omega_d(t)$ to generate the desired control torques $\tau_d(t)$. For this purpose, one has the following assumptions:

- The total disturbance estimation, angular velocity estimation and its time derivative will be denoted by $\hat{\zeta}(t)$, $\hat{\omega}_d(t)$ and $\dot{\hat{\omega}}_d(t)$ respectively.
- The perturbation function $\zeta(\cdot)$ is a uniformly absolutely bounded disturbance, *i.e.*, $\sup_t \|\zeta(\cdot)\| = \|\zeta(\cdot)\|_\infty \le K_0$.
- $\hat{\omega}_d(t)$ is uniformly absolutely bounded first-order time derivative, $\dot{\hat{\omega}}_d(t)$, *i.e.*, $\sup_t \|\hat{\omega}_d(t)\| \le K_0$,$\sup_t \|\dot{\hat{\omega}}_d(t)\| \le K_1$.

The attitude ADRC controller is structured by three blocks, the error quaternion, the bounded attitude control and the disturbance observer. First, the disturbance observer will be defined, consider the endogenously disturbed dynamic system (6), grouping endogenous terms dependent on the state $-[\omega^\times]J\omega + G_a$, and the exogenous

term dependent on time $\xi(t)$, we obtain the global perturbation of the system as $\zeta(t) = -[\omega^{\times}]J\omega + G_a + \xi(t)$, with uniformly absolutely bounded first and second time derivatives, *i.e.*, $\sup_t |\zeta_{d_i}(\cdot)| \leq K_0$. Furthermore, let K_{0_i} denote the bound for the disturbance function about the i^{th} axis.
Let's consider the following ESO:

$$\Sigma_{ESO} := \begin{cases} \dot{\hat{\omega}} = J^{-1}\left(\tau + \hat{\zeta}\right) + \Lambda_3\left(\omega - \hat{\omega}\right) \\ \dot{\hat{\zeta}} = \Lambda_2\left(\omega - \hat{\omega}\right) \end{cases} \tag{8}$$

where $\hat{\zeta}$, and $\dot{\hat{\zeta}}$ are the disturbance estimation and its time derivative, respectively, and τ is the control input. The set of matrices $\Lambda_3 = (l_3, l_3, l_3)$ and $\Lambda_2 = (l_2, l_2, l_2)$ are selected with the assistance of a desired closed-loop Hurwitz polynomial of third-order. Given a positive constant a, a continuous, nondecreasing function

$sat(x_k, a) : \mathbb{R} \to \mathbb{R}$ is defined by

$$sat(x_k, a) = \begin{cases} x_k, & x_k \leq a \\ a \cdot sign(x_k), & x_k > a \end{cases} \tag{9}$$

Assuming a sufficient time for $\zeta(t)$ to be estimated via ESO (**??**), the following attitude trajectory tracking control law is proposed.

Proposition 1 *Consider the rigid body rotational dynamics described by* (6) *with the following bounded control input vector* τ_d *such that*

$$\tau_d = -sat_{\tau_M}\left(f_{sat,N}(\tilde{q}, \tilde{\omega}) + \hat{\zeta}(t)\right) \tag{10}$$

where $f_{sat,N}$ *is defined as :*

$$f_{sat,N}(\tilde{q}, \tilde{\omega}) = -sat_N\left(\bar{\lambda}[\tilde{\omega} + \rho\tilde{q}]\right) \tag{11}$$

with $i \in \{1, 2, 3\}$ *and where* $\sigma_{M_{i1}}$ *and* $\sigma_{M_{i2}}$ *are saturation functions.*

4 Results

This section presents the results obtained. The problem of canceling the global disturbance (endogenous and exogenous) by means of an extended state observer ESO is addressed. Subsequently,a module dedicated to the generation of aggressive trajectories is designed. This module generates $\omega_{y,d}(t)$ time-varying trajectory to cause a simultaneous double turn around the axis VANT-UAV. The maneuver is accompanied by aerodynamic phenomena such as turbulence, external forces, ground effects and parametric uncertainties. The physical parameters of the VANT-UAV used are g: $9.81 m/s^2$,the mass m: 1.033 kg, and the diagonal matrix of inertia J:0.00653 kg.

4.1 Exogenous and Endogenous Disturbance Cancelation

As mentioned above, the global disturbance is generated only on the axis y_e^b, since this is the axis where the desired reference is generated. The global disturbance $\zeta(t)$ has been modeled as a uniformly bounded function varying in time (12).

$$\zeta(t) = 1.5 \cos 50t \sin 5t \sin 0.2t \tag{12}$$

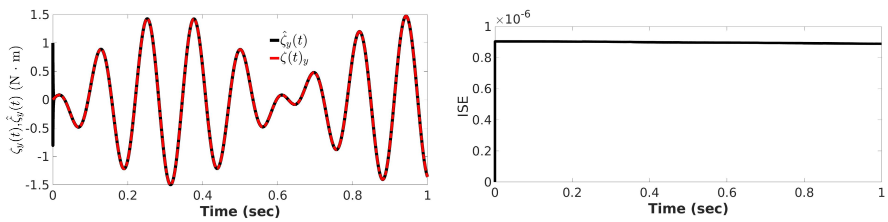

Fig. 2: Evolución del disturbio externo ζ_r y su estimación y_2.

The ESO (8) has been used with the following parameters, with $\Lambda_3 = 2\xi\omega_n/\epsilon^2$ and $\Lambda_2 = \omega_n^2/\epsilon$ where ξ, $w_n > 0$ and ϵ is a small parameter. As can be seen in Fig. 2 the value of the integral square error ISE(t) of the total disturbance estimate (13) indicates a constant value very close to zero, which indicates a rapid estimate of the global disturbance $\hat{\zeta}_y$ and redundant estimate angular velocity $\hat{\omega}_y$.

$$ISE(t) = \int_0^t \left[(\omega_y(\sigma) - \hat{\omega}_y(\sigma))^2 + (\zeta_y(\sigma) - \hat{\zeta}_y(\sigma))^2 \right] d\sigma \tag{13}$$

4.2 Trajectory Generation and Control of Multiple-Flips

In particular we address the problem of multiple flips around an axis of the VTOL-UAV. As shown in Fig. 3, the trajectory is divided into five states: elevation, increase, top, decrease and hover. The the desired angular velocity $\omega_d(t)$ that makes the drone rotate on the axis and is defined by the following equation:

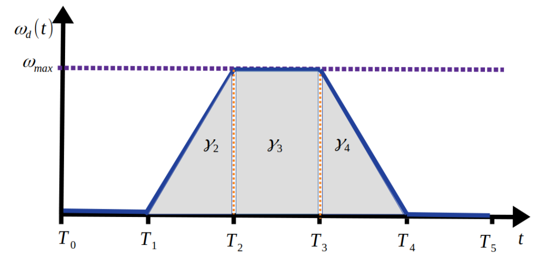

Fig. 3: UAV-VTOL trajectory tracking, double flip.

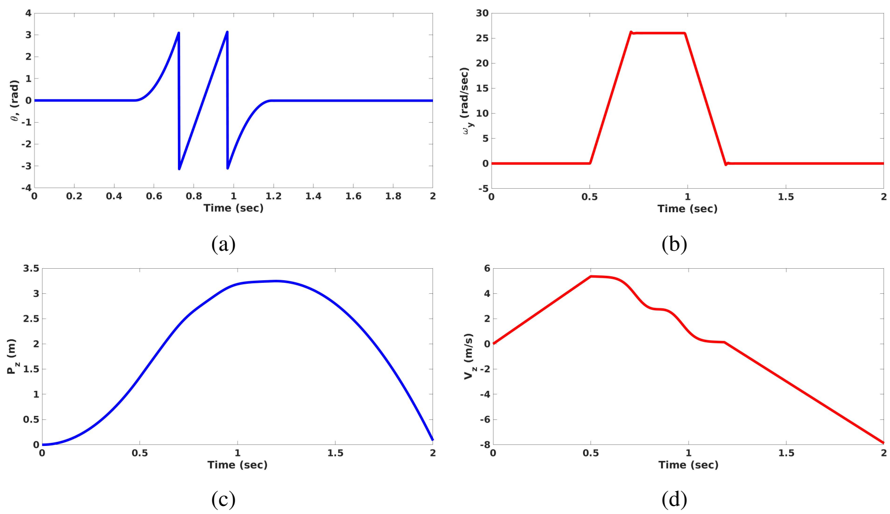

Fig. 4: Evolution of UAV-VTOL states. (a) attitude evolution, a conversion is made to Euler's angles for easy interpretation, (b) angular velocity evolution, (c) position evolution and (d) linear velocity evolution.

$$\omega_d(t) = \begin{cases} 0 & T_0 \leq t < T_1 \\ \frac{\omega_{max}}{\Delta_2}(t - T_0) & T_1 \leq t < T_2 \\ \omega_{max} & T_2 \leq t < T_3 \\ -\frac{\omega_{max}}{\Delta_4}(t - T_3) + \omega_{max} & T_3 \leq t < T_4 \\ 0 & T_4 \leq t < T_5 \end{cases} \tag{14}$$

where $\Delta T_i = T_{i-1} - T_i$. Considering the case of a double flip, the following parameters are obtained: $\omega_{max} = 26rad/sec$, $T_0 = 0$, $T_1 = 0.5$, $T_2 = 0.7$,$T_3 = 0.0983$,$T_4 = 1.183$, $T_5 = 1.6$.Considering the case of a double flip, the following parameters are obtained: $\omega_{max} = 26rad/sec$, $T_0 = 0$, $T_1 = 0.5$, $T_2 = 0.7$,$T_3 = 0.0983$,$T_4 = 1.183$, $T_5 = 1.6$.

The figure 4 shows the evolution of the attitude (euler angles), the angular velocity, altitude and linear velocity in the z axis. The control algorithm follows the reference despite external disturbances and uncertain parameters. There is no oscillation and the response is smooth and bounded.

5 Conclusions

The control law presented shows robustness to endogenous and exogenous shocks. Due to its simplicity, the control strategy can be implemented in computer systems with low computational power. The control is robust enough to follow trajectories to generate aggressive maneuvers with high precision

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