Fuzzy Conditional Inference and Reasoning for Fuzzy Granular Propositions using Two Fold Fuzzy Logic

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Abstract

In this paper, we consider fuzzy granular propositions with two fold fuzzy sets. We proposed fuzzy intuitions similar to Fukami intuitions. We studied fuzzy inference and reasoning for granular propositions using two fold fuzzy logic. We considered fuzzy granular intuitions containing "if $\cdots$ then $\cdots$ ", "if $\cdots$ then $\cdots$ else $\cdots$ ", "if $\cdots$ and/or $\cdots$ then $\cdots$ ". We used our method which is different from the Zadeh and Mamdani methods. we have shown that the all fuzzy intuitions are satisfied with our method

Keywords: fuzzy logic, two fold fuzzy logic, fuzzy conditional inference, fuzzy reasoning, fuzzy granular propositions, fuzzy intuitions,

1. Introduction

Zadeh [9] and Mamdani [2] proposed fuzzy conditional inference with a single membership function. Fukami[3] developed logical constructs for fuzzy intuitions using Godel definition and Standard sequence methods.We considered fuzzy granular intuitions with two fold fuzzy set/ The two fold fuzzy set is defined as $\tilde{A}$ where fuzzy set has two membership functions $\tilde{A}$=$\{A_1, A_2\}$, Where A_1 support the information and A_2 is against the information.
For instance two fold fuzzy set is given by
$\tilde{A}$=$\{true, false\}$, $\{belied, disbelief\}$,$\{known, unknown\}$, $\{likely, unlikely\}$ etc

We consider different types of fuzzy inferences with two fold fuzzy sets

The evidence is granular if it consists of collection of propositions[11],
$E = \{g_1, g_2, ..., g_n\}$
$E = \{g_1, g_2, ..., g_n\}$

g_1= x_1 is A_1 is λ_1
g_2= x_2 is A_2 is λ_2
$\cdots$
g_1= x_n is A_n is λ_n
Suppose we have granular propositions
g_1= x is very ripe is likely
g_2= x is ripe is very likely
g_3= x is ripe is unlikely
The granular propositions with two fold fuzzy set is given by
g_1= x_1 is $\tilde{A}_1$ is λ_1
g_2= x_2 is $\tilde{A}_2$ is λ_2
$\cdots$
g_n= x_n is $\tilde{A}_n$ is λ_n

g= x is $\tilde{A}$ is λ
Where $\tilde{A_1}$ is two fold fuzzy set For instance fuzzy granularity may be given by
$\tilde{A}$=$\{true, false\}$, λ= true, very false etc,
fuzzy granular may be applied on respective membership functions
$\tilde{A}_\lambda$=$\{true, veryfalse\}$
$\tilde{A}$=$\{belied, disbelief\}$, λ= belief, more or less disbelief etc,
$\tilde{A}_\lambda$=$\{belied, moreorlessdisbelief\}$
$\tilde{A}$=$\{known, unknown\}$, λ= not known, very unknown etc,
$\tilde{A}_\lambda$=$\{notknown, unknown\}$
$\tilde{A}$=$\{likely, unlikely\}$, λ= likely, very unlikely etc,
$\tilde{A}_\lambda$=$\{likely, veryunlikely\}$
Fuzzy conditional inference for granular propositions may be given as

Type-1
If x is $\tilde{P}$ then y is $\tilde{Q}$ is λ
x is $\tilde{P}_1$ is λ

---

y is ?

If *apple* is *red* then *apple* is *ripe* is very true
apple is very *ripe*is true

---

y is ?

Type-2
If x is $\tilde{P}$ then y is $\tilde{Q}$ else y is $\tilde{R}$ is more true
x is $\tilde{P}_1$

---

y is ?

If *apple* is *ripe* then *apple* is *taste* else *apple* is *sour* is very true
apple is very *ripe* is very true

---

y is ?

Type-3
If x is $\tilde{P}$ and If x is $\tilde{P}$ and x is $\tilde{Q}$ or x is $\tilde{R}$ then y is $\tilde{S}$ is λ_1
x is $\tilde{P}_1$ and x is $\tilde{Q}_1$ or x is $\tilde{R}_1$ *is* λ_2

---

y is ?

If x is red or x is ripe and x is big then x is taste is very very true
x is red or x is ripe and x is very big is very true

---

y is ?

Causal Logic

Consider the causal logical inference [1]
Modus Ponens
$p \rightarrow q$
P

---

q
Modus Tollens
$p \rightarrow q$
q'

P'
Generalization
$p \vee q = p$

$p \lor q = q$

Specialization

$p \land q = p$

$p \land q = q$

Table 1: Causal logic.

Causal Logic	Proposition	Inference
Modus Ponens	x is p	y is q
Modus Ponens	x is not p	y is not q
Modus Tollens	y is q	x is p
Modus Tollens	y is not q	x is not p

Fuzzy plausibility

Plausibility theory will made inconsistent information into consistent.

Generalization

$p\vee$ q, μ =p, μ
$p\vee$ q, μ = q, μ

Specialization

$p\wedge$ q , μ=p, μ
$p\wedge$ q = q, μ

The inference is given using generalization and specialization

p$\wedge$ q $\vee$ r, μ=p$\vee$ q, μ = p, μ
p$\wedge$ q $\vee$ r, μ=q$\vee$ r, μ = q, μ
p$\wedge$ q $\vee$ r, μ=r$\vee$ p, μ = p, μ

2. Two fold fuzzy logic

The fuzzy logic is combination of fuzzy sets using logical operators. The fuzzy logic with two fold fuzzy sets is combination of two fold fuzzy sets using logical operators. The fuzzy logic bases on two fold fuzzy set shall be studied similar lines of Zadeh fuzzy logic.

Some of the logical operations are given below for fuzzy sets with two fold fuzzy membership functions.

$\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are fuzzy sets with two fold fuzzy membership functions.

Negation

x is not $\tilde{A}$
$\tilde{A}'(x) = \{1 - \mu_A^{true}(x), 1 - \mu_A^{false}(x)\}/x$

Disjunction

x is $\tilde{A}$ or y is $\tilde{B}$
$\tilde{A} \vee \tilde{B} = \{\max(\mu_A^{true}(x), \mu_B^{true}(y)), \max(\mu_A^{false}(x), \mu_B^{false}(y))\}/(x,y),$

Conjunction
x is $\tilde{A}$ and y is $\tilde{B}$
$\tilde{A} \wedge \tilde{B} = \{\min(\mu_A^{true}(x), \mu_B^{true}(y)), \min(\mu_A^{false}(x), \mu_B^{false}(y))\}/(x,y),$

Composition
if x is $\tilde{A}$ then y is $\tilde{B}$
x is $\tilde{A}_1$
————————-
y is $\tilde{A}_1$ o $(\tilde{A} \rightarrow \tilde{B})$

$$\tilde{A} \text{ o } (\tilde{A} \rightarrow \tilde{B}) = \{\min\{\mu_A^{true}(x), \min(1, 1 - \mu_A^{true}(x) + \mu_B^{true}(y))\}, \\ \min\{\mu_A^{Diselief}(x), \min(1, 1 - \mu_A^{false}(x) + \mu_B^{false}(y)\}\}/y$$

if $x = y$
$$= \{\min\{\mu_A^{true}(x), \min(1, 1 - \mu_A^{true}(x) + \mu_B^{true}(x))\}, \\ \min\{\mu_A^{false}(x), \min(1, 1 - \mu_A^{false}(x) + \mu_B^{false}(x))\}$$

Fuzzy quantifiers
The fuzzy propositions may contain quantifiers like “very” , “more or less” etc. These fuzzy quantifiers may be eliminated as

Concentration
x is very $\tilde{A}$
$\mu_{very\ \tilde{A}}(x) = \{\mu_{very\ A}^{true}(x)^2, \mu_{very\ A}^{false}(x)^2\}$

Diffusion
if x is more or less $\tilde{A}$
$\mu_{more\ or\ less\ \tilde{A}}(x) = \{\mu_{more\ or\ less\ A}^{true}(x)^2, \mu_{more\ or\ less\ A}^{false}(x)^{0.5}\}$

3. Fuzzy Conditional Inference for Fuzzy Granular Propositions

The fuzzy conditional propositions is of the form ”if (precedent part) then (consequent part)”.

The consequent part is derived from precedent part for fuzzy conditional inference [6].
$\int \mu_Q(y) = \int \mu_P(x)$, i.e., $Q \subseteq P$ and $P \subseteq Q$

Mamdani [2] fuzzy conditional inference is given by if x is P then y is Q= $\{\int \mu_P(x)\times \int \mu_Q(x)\ \}$

if x is P then y is Q= $\{\int \mu_P(x)\ \}$

Consider the fuzzy conditional inference
If x is $\tilde{P}$ then y is $\tilde{Q}$ else y is $\tilde{R}$

The fuzzy conditional inference is given by

If x is $\tilde{P}$ then y is $\tilde{Q}$ $\vee$ If x is $\tilde{P\prime}$ then y is $\tilde{R}$

Using Fuzzy plausibility, fuzzy inference is given by

If x is $\tilde{P}$ then y is $\tilde{Q}$
If x is $\tilde{P\prime}$ then y is $\tilde{R}$

Confider fuzzy inference Type-1
The fuzzy inference is given for Type-1 is given by using generalization and specialization

If x is $\tilde{P}_\lambda$ then y is $\tilde{Q}_\lambda$ is λ
x is $\tilde{P}_1$ is λ

---

y is $\tilde{Q}_{1\lambda}$

Confider fuzzy inference Type-2
The fuzzy inference is given for Type-2 is given by using generalization and specialization

The fuzzy inference is given for Type-2 using generalization and specialization

if x is $\tilde{P}_\lambda$ then x is $\tilde{Q}_\lambda$
x is $\tilde{P}_1$

---

y is $\tilde{Q}_{1\lambda}$

if x is P' then x is $\tilde{R}_\lambda$
x is $\tilde{P}_1$

---

y is $\tilde{R}_{1\lambda}$

Confider fuzzy inference Type-3

The fuzzy inference is given for Type-3 is given by using generalization and specialization

If x is $\tilde{P}_\lambda$ then y is $\tilde{S}$
x is $\tilde{P}_1$

---

y is $\tilde{S}_1$

If x is $\tilde{Q}_\lambda$ then y is $\tilde{S}$
x is $\tilde{Q}_1$

---

y is $\tilde{S}_\lambda$

If x is $\tilde{R}_\lambda$ then y is $\tilde{S}$
x is $\tilde{R}_1$

---

y is $\tilde{S}_{1\lambda}$

Type-1, Type-2 and Type-3 will give two criterions.

Criteria-1$_\lambda$
If x is $\tilde{P}_\lambda$ then y is $\tilde{Q}_\lambda$
x is $\tilde{P}_{1\lambda}$

---

y is $\tilde{Q}_{1\lambda}$

Type-11, Type-21 and Type-31 gives two criterions

Criteria-2$_\lambda$
If x is $\tilde{P'}_\lambda$ then y is $\tilde{R}_\lambda$
x is $\tilde{P}'_{1\lambda}$

---

y is $\tilde{R}_{1\lambda}$

The fuzzy intuitions for Criteria-1$_\lambda$ are given by

4. Verification of fuzzy intuition using Fuzzy Conditional Inference

5. Fuzzy Granular Propositions

The granular fuzzy inference is drawing a conclusion from granular fuzzy propositions.

Table 4: Fuzzy inference for Criteria-1_λ.

Intuition	Proposition	Inference
$I1_\lambda$	x is $\tilde{P}_\lambda$	y is $\tilde{Q}_\lambda$
$I2_\lambda$	y is $\tilde{Q}_\lambda$	x is $\tilde{P}_\lambda$
$II1_\lambda$	x is very $\tilde{P}_\lambda$	y is very $\tilde{Q}_\lambda$
$II1_\lambda$	y is very $\tilde{Q}_\lambda$	x is very $\tilde{P}_\lambda$
$III1_\lambda$	x is more or less $\tilde{P}_\lambda$	y is more or less $\tilde{Q}_\lambda$
$III2_\lambda$	y is more or less $\tilde{Q}_\lambda$ o λ	is more or less $\tilde{P}_\lambda$
$IV1_\lambda$	x is not $\tilde{P}_\lambda$	y is not $\tilde{Q}_\lambda$
$IV2_\lambda$	y is not $\tilde{Q}_\lambda$	x is not $\tilde{P}_\lambda$

Fuzzy Conditional Inference shall be verified similar lines of Criteria-1

Verification of fuzzy intuitions for Criteria-1

4.1.1 In the case of intuition I1

$\tilde{P}_\lambda$ o ($\tilde{P}_\lambda \rightarrow \tilde{Q}_\lambda$)
$= \int \mu_{\tilde{P}}(x)$ o $(\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$
Considering $\tilde{P}_\lambda \rightarrow \tilde{S} = \tilde{P}$
Considering $\tilde{S} = \tilde{P}$
$= \int \mu_{\tilde{S}}(y)$ o $(\int \mu_{\tilde{S}}(y))$
$= \int \mu_{\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$
Using specialization
$= \int \mu_{\tilde{S}}(y)$
$= y$ is $\tilde{\tilde{S}}$
intuition I-1 satisfied.

4.1.2 In the case of intuition I2

$(\tilde{P}_\lambda \rightarrow \tilde{Q}_\lambda)$ o $\tilde{Q}_\lambda$
$= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$ o $\int \mu_{\tilde{S}}(y)$
Considering $\tilde{P}_\lambda \rightarrow \tilde{S} = \tilde{P}$
Considering $\tilde{S} = \tilde{P}$
$= \int \mu_{\tilde{P}}(x))$ o $\int \mu_{\tilde{P}}(x)$

$= \int \mu_{\tilde{P}}(x)) \wedge \int \mu_{\tilde{P}}(x)$

Using specialization

$= \int \mu_{\tilde{P}}(x)$

$= x$ is $\tilde{\tilde{P}}$

intuition I-2 satisfied.

4.1.3 In the case of intuition II1

$very\tilde{P}$ o $(\tilde{P}_{\lambda} \rightarrow \tilde{Q}_{\lambda})$

$= \int \mu_{very\tilde{P}}(x)$ o $(\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$

Considering $\tilde{P}_{\lambda} \rightarrow \tilde{S} = \tilde{P}$

Considering $\tilde{S} = \tilde{P}$

$= \int \mu_{very\tilde{S}}(y)$ o $(\int \mu_{\tilde{S}}(x))$

$= \int \mu_{very\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$

Using specialization

$= \int \mu_{very\tilde{S}}(y)$

$= y$ is $very\tilde{S}$

intuition II-1 satisfied.

4.1.4 In the case of intuition II2

$(\tilde{P}_{\lambda} \rightarrow \tilde{Q}_{\lambda})$ o very $\tilde{Q}_{\lambda}$

$= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{very\tilde{S}}(y))$ o $\int \mu_{\tilde{S}}(y)$

Considering $\tilde{P}_{\lambda} \rightarrow \tilde{S} = \tilde{P}$

Considering $\tilde{S} = \tilde{P}$

$= \int \mu_{\tilde{P}}(x))$ o $\int \mu_{very\tilde{P}}(x)$

$= \int \mu_{\tilde{P}}(x)) \wedge \int \mu_{very\tilde{P}}(x)$

Using specialization

$= \int \mu_{very\tilde{P}}(x)$

$= x$ is $very\tilde{P}$

intuition II-2 satisfied.

4.1.5 In the case of intuition III1

indent $moreorless\tilde{P}$ o $(\tilde{P}_{\lambda} \rightarrow \tilde{Q}_{\lambda})$

$= \int \mu_{moreorless\tilde{P}}(x)$ o $(\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$

Considering $\tilde{P}_{\lambda} \rightarrow \tilde{S} = \tilde{P}$

Considering $\tilde{S} = \tilde{P}$

$= \int \mu_{moreorless\tilde{S}}(y)$ o $(\int \mu_{\tilde{S}}(x))$

$= \int \mu_{moreorless\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$

Using specialization

$= \int \mu_{moorless\tilde{S}}(y)$

$=y$ is $\widetilde{very \tilde{S}}$
intuition III-1 satisfied.

4.1.6 In the case of intuition III2

$(\tilde{P}_\lambda \rightarrow \tilde{Q}_\lambda)$ o more or less $\tilde{Q}_\lambda$
$= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\underset{\sim}{moreorless}\tilde{S}}(y))$ o $\int \mu_{\tilde{S}}(y)$
Considering $\tilde{P}_\lambda \rightarrow \tilde{S} = \tilde{P}$
Considering $\tilde{S} = \tilde{P}$
$= \int \mu_{\tilde{P}}(x))$ o $\int \mu_{moreorless\tilde{P}}(x)$
$= \int \mu_{\tilde{P}}(x)) \wedge \int \mu_{moreorless\tilde{P}}(x)$
Using specialization
$= \int \mu_{\underset{\sim}{moreorless}\tilde{P}}(x)$
$=x$ is $very\tilde{P}$
intuition III-2 satisfied.

4.1.7 In the case of intuition IV1

not $\tilde{P}_\lambda$ o $(\tilde{P}_\lambda \rightarrow \tilde{\tilde{S}})$
$= \int \mu_{not\tilde{P}}(x)$ o $(\int \mu_{\underset{\sim}{\tilde{P}}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$
Considering $\tilde{P}_\lambda \rightarrow \tilde{S} = \tilde{P}$
Considering $\tilde{S} = \tilde{P}$
$= \int \mu_{not\tilde{S}}(y)$ o $(\int \mu_{\tilde{S}}(x))$
$= \int \mu_{not\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$
Using specialization
$= \int \mu_{\underset{\sim}{not\tilde{S}}}(y)$
$=y$ is $not\tilde{S}$
intuition IV-1 satisfied.

4.1.8 In the case of intuition IV2

$(\tilde{P}_\lambda \rightarrow \tilde{Q}_\lambda)$ o very $\tilde{Q}_\lambda$
$= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\underset{\sim}{very}\tilde{S}}(y))$ o $\int \mu_{\tilde{S}}(y)$
Considering $\tilde{P}_\lambda \rightarrow \tilde{S} = \tilde{P}$
Considering $\tilde{S} = \tilde{P}$
$= \int \mu_{\tilde{P}}(x))$ o $\int \mu_{very\tilde{P}}(x)$
$= \int \mu_{\tilde{P}}(x)) \wedge \int \mu_{very\tilde{P}}(x)$
Using specialization
$= \int \mu_{\underset{\sim}{very\tilde{P}}}(x)$
$=x$ is $very\tilde{P}$
intuition IV-2 satisfied.

Criteria-1 is suitable for I1,I-, II1, II2, III1, III2, IV1 and IV2.

Criteria-1_λ will be satisfies for $I1_\lambda$,$I2_\lambda$, $II1_\lambda 1$, $II2_\lambda$, $III1_\lambda$, $III2_\lambda$, $IV1_\lambda$ and $IV2_\lambda$.

The fuzzy intuitions for Criteria-2_λ are given by

Table 5: Fuzzy inference for Criteria-2_λ.

Intuition	Proposition	Inference
$I1'_\lambda$	x is $\tilde{P'}_\lambda$	y is $\tilde{R}_\lambda$
$I2'_\lambda$	y is $\tilde{R}_\lambda$	x is $\tilde{P'}_\lambda$
$II1'_\lambda$	x is very $\tilde{P'}_\lambda$	y is very $\tilde{R}_\lambda$
$II2'_\lambda$	y is very $\tilde{R}_\lambda$ o	x is very $\tilde{P'}_\lambda$
$III1'_\lambda$	x is more or less $\tilde{P'}_\lambda$	y is more or less $\tilde{R}_\lambda$
$III2'_\lambda$	y is More or less $\tilde{R}_\lambda$	is more or less $\tilde{P}._\lambda$
$IV1'_\lambda$	x is not $\tilde{P'}_\lambda$	y is not $\tilde{R}_\lambda$
$IV2'_\lambda$	y is not $\tilde{R}_\lambda$ o is	x is not $\tilde{P'}_\lambda$

Fuzzy Conditional Inference shall be verified similar lines of Criteria-2
Verification of fuzzy intuitions for Criteria-2

4.2.1 In the case of intuition I'1

$\tilde{P}_\lambda$ o $(\tilde{P'}_\lambda \to \tilde{R}_\lambda)$
$= \int \mu_{\tilde{P}}(x)$ o $(\int \mu_{\tilde{P'}}(x) \to \int \mu_{\tilde{R}}(y))$
Considering $\tilde{P'}_\lambda \to \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{\tilde{R}}(y)$ o $(\int \mu_{\tilde{R}}(y))$
$= \int \mu_{\tilde{R}}(y) \wedge (\int \mu_{\tilde{R}}(y))$
Using specialization
$= \int \mu_{\tilde{R}}(y)$
$=y$ is $\tilde{\tilde{R}}$
intuition I'-1 satisfied.

4.2.2 In the case of intuition I'2

$(\tilde{P'}_\lambda \to \tilde{R}_\lambda)$o $\tilde{R'}$
$= (\int \mu_{\tilde{P'}}(x) \to \int \mu_{\tilde{R}}(y))$ o $\int \mu_{\tilde{R'}}(y)$
Considering $\tilde{P'}_\lambda \to \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{\tilde{P'}}(x))$ o $\int \mu_{\tilde{P'}}(x)$

$= \int \mu_{\tilde{P'}}(x)) \wedge \int \mu_{\tilde{P'}}(x)$
Using specialization
$= \int \mu_{\tilde{P'}}(x)$
$= x$ is $\tilde{P'}_\lambda$
intuition I'-2 satisfied.

4.2.3 In the case of intuition II'1

very $\tilde{P'}_\lambda$ o $(\tilde{P'}_\lambda \rightarrow \tilde{R}_\lambda)$
$= \int \mu_{very\tilde{P'}}(x)$ o $(\int \mu_{\tilde{P'}}(x) \rightarrow \int \mu_{\tilde{R}}(y))$
Considering $\tilde{P'}_\lambda \rightarrow \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{very\tilde{R'}}(y)$ o $(\int \mu_{\tilde{R}}(y))$
$= \int \mu_{very\tilde{R'}}(y) \wedge (\int \mu_{\tilde{R}}(y))$
Using specialization
$= \int \mu_{very\tilde{R}}(y)$
$= y$ is very $\tilde{R}_\lambda$
intuition II'-1 satisfied.

4.2.4 In the case of intuition II'2

$(\tilde{P'}_\lambda \rightarrow \tilde{R}_\lambda)$o very $\tilde{R}_\lambda$
$= (\int \mu_{\tilde{P'}}(x) \rightarrow \int \mu_{\tilde{R}}(y))$ o $\int \mu_{very\tilde{R}}(y)$
Considering $\tilde{P'}_\lambda \rightarrow \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{\tilde{P'}}(x))$ o $\int \mu_{very\tilde{P'}}(x)$
$= \int \mu_{\tilde{P'}}(x)) \wedge \int \mu_{very\tilde{P'}}(x)$
Using specialization
$= \int \mu_{very\tilde{P'}}(x)$
$= x$ is very $\tilde{P'}_\lambda$
intuition II'-2 satisfied.

4.2.5 In the case of intuition III'1

more or less $\tilde{P'}_\lambda$ o $(\tilde{P'}_\lambda \rightarrow \tilde{R}_\lambda)$
$= \int \mu_{moreorless\tilde{P'}}(x)$ o $(\int \mu_{\tilde{P'}}(x) \rightarrow \int \mu_{\tilde{R}}(y))$
Considering $\tilde{P'}_\lambda \rightarrow \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{moreorless\tilde{R'}}(y)$ o $(\int \mu_{\tilde{R}}(y))$
$= \int \mu_{moreorless\tilde{R}}(y) \wedge (\int \mu_{\tilde{R}}(y))$
Using specialization
$= \int \mu_{moreorless\tilde{R}}(y)$

$=y$ is more or less $\tilde{R}_\lambda$
intuition III'-1 satisfied.

4.2.6 In the case of intuition III'2

$(\tilde{P'}_\lambda \rightarrow \tilde{R}_\lambda)$ o more or less $\tilde{R}_\lambda$
$= (\int \mu_{\tilde{P'}}(x) \rightarrow \int \mu_{\tilde{R}}(y))$ o $\int \mu_{mororless\tilde{R}}(y)$
Considering $\tilde{P'}_\lambda \rightarrow \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{\tilde{P'}}(x))$ o $\int \mu_{moreorless\tilde{P'}}(x)$
$= \int \mu_{\tilde{P'}}(x)) \wedge \int \mu_{moreorless\tilde{P'}}(x)$
Using specialization
$= \int \mu_{moreorless\tilde{P'}}(x)$
$=x$ is more or less $\tilde{P'}_\lambda$
intuition II'-2 satisfied.

4.2.7 In the case of intuition IV'1

not $\tilde{P'}_\lambda$ o $(\tilde{P}_\lambda$' $\rightarrow \tilde{R}_\lambda)$
$= \int \mu_{not\tilde{P'}}(x)$ o $(\int \mu_{\tilde{P'}}(x) \rightarrow \int \mu_{\tilde{R}}(y))$
Considering $\tilde{P'}_\lambda \rightarrow \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{not\tilde{R'}}(y)$ o $(\int \mu_{\tilde{R}}(y))$
$= \int \mu_{not\tilde{R'}}(y) \wedge (\int \mu_{\tilde{R}}(y))$
Using specialization
$= \int \mu_{not\tilde{R}}(y)$
$=y$ is not $\tilde{R}_\lambda$

intuition IV'-1 satisfied.

4.2.8 In the case of intuition IV'2

$(\tilde{P'}_\lambda \rightarrow \tilde{R}_\lambda)$ o not $\tilde{R}_\lambda$
$= (\int \mu_{\tilde{P'}}(x) \rightarrow \int \mu_{\tilde{R}}(y))$ o $\int \mu_{not\tilde{R}}(y)$
Considering $\tilde{P'}_\lambda \rightarrow \tilde{R} = \tilde{P'}$
Considering $\tilde{R} = \tilde{P'}$
$= \int \mu_{\tilde{P'}}(x))$ o $\int \mu_{not\tilde{P'}}(x)$
$= \int \mu_{\tilde{P'}}(x)) \wedge \int \mu_{not\tilde{P'}}(x)$
Using specialization
$= \int \mu_{not\tilde{P'}}(x)$
$=x$ is not $\tilde{P'}_\lambda$
intuition IV'-2 satisfied.

Criteria-1 is suitable for I'1,I'2, II'1, II'2, III'1, III'2, IV'1 and IV'2.

Criteria-2_λ will be satisfies for $I1'_\lambda$,$I2'_\lambda$, $II1'_\lambda 1$, $II2'_\lambda$, $III1'_\lambda$, $III2'_\lambda$, $IV1'_\lambda$ and $IV2_\lambda$.

6. Granular Fuzzy Certainty Factor

The granular fuzzy certainty factor(GFCF) shall made as single fuzzy membership functions with two fuzzy membership functions to eliminate the conflict of evidence between "true "and "false".

Definition 4.1 *The FCF of $\mu_{\tilde{A}}$ for propositions "x is $\tilde{A}$" is characterized by its membership function $\mu_{\tilde{A}}^{FCF}(x) \to [0,1]$, where $\mu_{\tilde{A}}^{FCF}(x) = \{\mu_A^{true}(x) - \mu_A^{false}(x)\}/x$,*
$\mu_{\tilde{A}}^F CF(x) < 0, \mu_{\tilde{A}}^F CF(x) = 0$ *and* $\mu_{\tilde{A}}^F CF(x) > 0$
are the redundant, insufficient and sufficient respectively.

The FCF will become single fuzzy membership function.
The GFCF is give by
$\mu_{\tilde{A}}^{FCF}(x) o \lambda$
$= \{\mu_A^{true}(x) - \mu_A^{false}(x)\} o \lambda$
For instance
$\mu_{\tilde{A}}^{FCF}(x) o$very true
$= \{\mu_A^{true}(x) - \mu_A^{false}(x)\} o$very true
$= \{\mu_A^{true}(x)^2 - \mu_A^{false}(x)\}$

7. Conclusion

Fuzzy intuitions and Fuzzy granular intuitions are considered for two fold fuzzy logic. These intuitions are studied for fuzzy conditional inference containing "and/or "and "if $\cdots$ then $\cdots$ else $\cdots$ ". All fuzzy intuitions are satisfied with our fuzzy conditional inference method which is different from Zadeh and Mamdani fuzzy conditional inference methods. Fuzzy granular certainty factor is made as a single membership function to eliminate conflict between two membership functions.

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