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# Diagram of concrete dynamic deformation confined by CFRP jackets

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**Abstract.** A review of the current state of the issue of describing the deformation of columns confined by CFRP jackets at of dynamic loading is carried out. The insufficiency of the study of elements at the influence of axial dynamic load is indicated. The dynamic increase factors (DIF) for concrete and CFRP, which are obtained by the results of existing experiments, are substantiated. Using the principle of the invariance of the potential energy of deformation of concrete at the time of its destruction under regime loading, a formula is obtained for determining DIF for ultimate relative strain of unconfined concrete. Based on the assumption of the same law of concrete deformation at static and dynamic loading, was obtained a diagram of concrete dynamic deformation confined by CFRP jackets. The diagram is valid in the range of strain rates  $10^{-3}$ - $10^2$  s<sup>-1</sup>. A comparison is made between a static diagram and a dynamic diagram derived from it. The characteristic of the main sections of the diagram is given. An increase in the strength and ductility of confined concrete at the initial stage of loading is obtained. At stresses equal to the tensile strength of limited concrete, its ductility is somewhat reduced. It was revealed that a significant increase in the bearing capacity of confined concrete begins at strain rates of 10 s<sup>-1</sup> or more.

## 1. Introduction

Currently, composite materials are most in demand when reinforcing reinforced concrete structures damaged as a result of accidents, which allows them to restore their original bearing capacity [1]. However, the method is also applicable to withstand accidental impacts, that is, amplification is carried out before an emergency occurs. Most often, such situations are accompanied by dynamic loads.

The study of reinforced concrete deformation diagrams in the development of methods for calculating structures for dynamic effects is quite an urgent task. When describing diagrams at high loading speeds, the specific behavior of not only individual materials, but also the system as a whole, should be taken into account.

There are many publications devoted to the study of the operation of elements strengthening by CFRP jackets at high strain rates. Most of these works focus on emergency shock loads in the horizontal direction, for example, during explosions and collisions with vehicles [2, 3].

The technology of reinforcing columns with composite materials is quite relevant in seismically dangerous areas, due to the low weight of the strengthening elements. This explains the ongoing work on the study of structures strengthening by CFRP jackets at seismic loading [4].

Not many publications are devoted to the study of the compressed columns confined by CFRP jackets under axial dynamic loading. This loading scenario is quite common when performing building calculations for progressive destruction [5].

In the case of confined concrete by transverse steel reinforcement, this issue has been studied in sufficient detail [6, 7].

The work [8] is devoted to the analysis of the operation of elements strengthening by CFRP jackets under dynamic loading. A method for determining the bearing capacity of elements based on a nonlinear deformation model is developed. The technique is consistent with experimental data.

The development of a dynamic deformation diagram of confined concrete by CFRP jackets remains a rather urgent task. In this paper, an attempt is made to describe such a diagram by modifying the corresponding deformation diagram at quasistatic loading.

## 2. Quasistatic diagram

To describe the deformation diagram of concrete confined by CFRP jackets enough dependencies have been proposed [9, 10, 11], which have received experimental confirmation. In this paper, the diagram obtained in the study [9] is taken as the basis. The diagram has two characteristic sections: parabolic, at the initial stage of concrete deformation before significant damage to its structure occurs; and a rectilinear section characterizing the work of concrete in a pseudoplastic state [12], when only CFRP jacket resists to the development of transverse deformations. The diagram is written as a system of equations

$$\sigma_c = \begin{cases} E_c \varepsilon_c - \frac{(E_c - E_2)}{4f_{co}} (\varepsilon_c)^2, & 0 \leq \varepsilon_c \leq \varepsilon_t \\ f_{co} + E_2 \varepsilon_c, & \text{if } \rho_K \geq 0.01 \\ f_{co} - \frac{f_{co} - f_{cu}}{\varepsilon_{cu} - \varepsilon_{co}} (\varepsilon_c - \varepsilon_{co}), & \text{if } \rho_K < 0.01 \end{cases} \quad \varepsilon_t < \varepsilon_c \leq \varepsilon_{cu} \quad (1)$$

where  $\sigma_c$ ,  $\varepsilon_c$  – respectively, current compressive stresses and strains;

$f_{co}$ ,  $\varepsilon_{co}$ ,  $E_c$  – ultimate strength, ultimate relative deformations and initial elastic modulus of unconfined concrete, respectively;

$f_{cc}$ ,  $\varepsilon_{cu}$  – ultimate strength and ultimate relative deformations of confined concrete, respectively;

$E_2$  – elastic modulus of confined concrete on a linear section of the deformation diagram, determined by the formula

$$E_2 = \frac{f_{cc} - f_{co}}{\varepsilon_{cu}} \quad (2)$$

Values  $f_{cc}$  and  $\varepsilon_{cu}$  determined from the conditions

$$\frac{f_{cc}}{f_{co}} = 1 + 3,5(\rho_K - 0,01)\rho_\varepsilon \quad (3)$$

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1,75 + 6,5\rho_K^{0,8}\rho_\varepsilon^{1,45} \quad (4)$$

where  $\rho_K$  – confinement stiffness ratio;

$\rho_\varepsilon$  – strain ratio;

$$\rho_K = \frac{2E_f t_f}{\frac{f_{co}}{\varepsilon_{co}} D} \quad (5)$$

$$\rho_\varepsilon = \frac{\varepsilon_f}{\varepsilon_{co}} \quad (6)$$

where  $\varepsilon_f$  – ultimate strain of the composite material equal to

$$\varepsilon_f = \frac{R_f}{E_f} \quad (7)$$

where  $R_f$ ,  $E_f$  – ultimate strength and modulus of elasticity of a CFRP, respectively. The strains corresponding to the transition from the parabolic section of the diagram to the linear one are determined by the formula

$$\varepsilon_t = \frac{2f_{co}}{E_c - E_2} \quad (8)$$

It is worth noting that with the  $E_2 = 0$ , the first expression of system (1) will describe the deformation of unlimited concrete in accordance with the diagram proposed in [13]. Taking the value of ultimate relative strains  $\varepsilon_{co} = 0.002$ , from expression (8) for unconfined concrete we get

$$0.002 = \frac{2f_{co}}{E_c} \quad (9)$$

Where do we find  $E_c = 1000f_{co}$ , that is somewhat different from  $E_c = 4730\sqrt{f_{co}}$ , that proposed in [9].

To set the deformation diagram of concrete confined by CFRP jackets under dynamic loading, we will use the approach used in [6, 14]. Suppose that the deformation of concrete strengthening by CFRP jackets at dynamic loading obeys the same laws as at static loading. In this case, the parameters describing the deformation diagram  $f_{co}$ ,  $\varepsilon_{co}$ ,  $E_c$ ,  $E_f$  and  $\varepsilon_f$ , are modified depending on the deformation rate by introducing the corresponding dynamic increase factors (DIF). To indicate the parameters of the diagram at dynamic loading, the superscript “d” is used.

### 3. Dynamic increase factors of concrete

The strength properties of concrete under the influence of dynamic load increase, while its plasticity decreases with increasing strain rate. The increased dynamic strength of concrete is associated with the appearance of inertial forces of viscous resistance, which control the development of transverse deformations.

To describe the behavior of concrete at the influence of dynamic loads, many dependencies have been proposed [6, 15, 16, 17]. As studies show, the DIF of concrete depends not only on the strain rate, but also on the type of stress state [16], and on concrete strengths [15]. However, regarding the latter, researchers have no consensus. In the work [17] it is noted that, in view of the dimensionless nature of DIF, the results obtained on concrete of one class are applicable to others.

Dynamic loads on building structures most often occur as a result of accidents, such as fires. In such cases, the DIF of concrete will be a function of not only the strain rate, but also the temperature [18, 19].

The DIF of concrete is determined by the following expression

$$k_{d,c}^f = \frac{f_{co}^d}{f_{co}} \quad (10)$$

To describe DIF, we use the logarithmic dependence [17], which is valid at strain rates  $\dot{\varepsilon} \leq 10^2 \text{ s}^{-1}$ .

$$k_{d,c}^f = 1 + (\lg \varepsilon + 3) \cdot 0.03438 \quad (11)$$

Studies show that concrete deformation in the elastic stage does not depend on the loading rate, so the initial modulus of elasticity under static and dynamic compression are most often assigned equal to each other [15]. In addition, it was indicated in the work [16] that this type of deformation will also be valid for the pseudoplastic stage, characterized by intensive growth and opening of macrocracks. However, the influence of the strain rate is manifested in the section of the diagram, which is limited by strains corresponding to the lower and upper limit of crack formation. Taking this fact into account is of little interest from a practical point of view, since it will only affect the shape of the curved section of the confined concrete diagram and will not affect the ultimate stresses and strains.

In this work, used equality between the initial modulus of elasticity at the static and dynamic action of the load, i.e.

$$E_c^d = E_c \quad (12)$$

As noted, the ultimate strain on concrete decreases with increasing loading speed. We introduce the DIF for ultimate strain of concrete, similarly to the dependence (10)

$$k_{d,c}^\varepsilon = \frac{\varepsilon_{co}^d}{\varepsilon_{co}} \quad (13)$$

To determine the ultimate relative strains at dynamic loading, we use the postulate on the invariance of the potential energy of deformation of concrete at the time of its destruction, which was first proposed in [20]. It is considered that the work done on concrete at the time of destruction is constant, regardless of the strain rate. This approach was applied to concrete with indirect transverse steel reinforcement in work [6]. The postulate is given in the form

$$\int_0^{\varepsilon_{co}} \sigma_c d\varepsilon_c = \int_0^{\varepsilon_{co}^d} \sigma_c^d d\varepsilon_c^d \quad (14)$$

To describe the deformation diagram of unconfined concrete on an ascending branch, as noted earlier, we use the first expression of system (1) for  $E_2 = 0$ . Performing the substitution in (14), integrating and giving similar ones, we obtain

$$\frac{E_c^d}{6f_{co}^d} (\varepsilon_{co}^d)^3 - (\varepsilon_{co}^d)^2 + \varepsilon_{co}^2 \left(1 - \frac{E_c}{6f_{co}} \varepsilon_{co}\right) = 0 \quad (15)$$

With this in mind (13), as well as with  $\varepsilon_{co} = 0.002$  and  $E_c = 1000f_{co}$ , write

$$(k_{d,c}^\varepsilon)^3 - 3k_{d,c}^f (k_{d,c}^\varepsilon)^2 + 2k_{d,c}^f = 0 \quad (16)$$

Finding the root of the cubic equation at a point  $(k_{d,c}^f; k_{d,c}^\varepsilon) = (1; 1)$ , we obtain the formula for determining DIF for ultimate relative strains.

$$k_{d,c}^\varepsilon = k_{d,c}^f \left[ 1 - 2 \cos \left( \frac{1}{3} \arccos \left( \frac{1}{(k_{d,c}^f)^2} - 1 \right) - \frac{2\pi}{3} \right) \right] \quad (17)$$

#### 4. Dynamic increase factors of CFRP

In contrast to concrete in CFRP at dynamic loading, not only the strength characteristics increase, but also the ultimate tensile strain and modulus of elasticity [21, 22, 23]. In this case, a linear stress-strain relation is maintained.

It is noting that when exposed to dynamic loads, the risk of delamination of the composite material along the adhesive joint or on contact with the concrete surface increases, as well as damage to the structure of the matrix, rupture of individual fibers and delamination into separate layers [24]. Such destruction scenarios should be taken into account by appropriate factor of safety. However, it can be assumed that the occurrence of failure due to peeling of the composite during a single dynamic loading

without unloading is unlikely, since even in the absence of adhesion to the concrete surface, jacket will resist the development of deformations in the transverse direction.

For describe the change in the mechanical characteristics of the CFRP depending on the strain rate, we use the dependencies proposed in work [21], which were tested in the range of strain rates  $2.42 \cdot 10^{-4} \leq \dot{\varepsilon} \leq 87.4 (\approx 10^2) \text{ s}^{-1}$

$$R_f^d = R_f (1 + 4.496 \cdot 10^{-4} \dot{\varepsilon}^{1.529}) \quad (18)$$

$$E_f^d = E_f (1 + 5.634 \cdot 10^{-3} \dot{\varepsilon}^{0.8228}) \quad (19)$$

$$\varepsilon_f^d = \varepsilon_f (1 + 5.723 \cdot 10^{-7} \dot{\varepsilon}^{2.89}) \quad (20)$$

Analyzing formulas (18)-(20), it can be noted that the values do not quite correctly correspond to the linear relationship between stresses and strains; therefore, it is recommended to replace formula (20) with

$$\varepsilon_f^d = \frac{R_f^d}{E_f^d} \quad (21)$$

In this case, it is expression (20) that is replaced, and not (18), since the initial values for designing are most often the values of  $R_f$  and  $E_f$ .

## 5. Dynamic diagram

The diagram of the deformation of a compressed concrete confined by CFRP jacket at dynamic loading can be represented as the following set of equations

$$\sigma_c^d = \begin{cases} E_c \varepsilon_c^d - \frac{(E_c - E_2^d)}{4 f_{co}^d} (\varepsilon_c^d)^2, & 0 \leq \varepsilon_c^d \leq \varepsilon_t^d \\ f_{co}^d + E_2 \varepsilon_c^d, & \text{if } \rho_\kappa^d \geq 0.01 \\ f_{co}^d - \frac{f_{co}^d - f_{cu}^d}{\varepsilon_{cu}^d - \varepsilon_{co}^d} (\varepsilon_c^d - \varepsilon_{co}^d), & \text{if } \rho_\kappa^d < 0.01 \end{cases} \quad \varepsilon_t^d < \varepsilon_c^d \leq \varepsilon_{cu}^d \quad (22)$$

where

$$E_c^d = E_c \quad (23)$$

$$f_{co}^d = k_{d,c}^f f_{co} \quad (24)$$

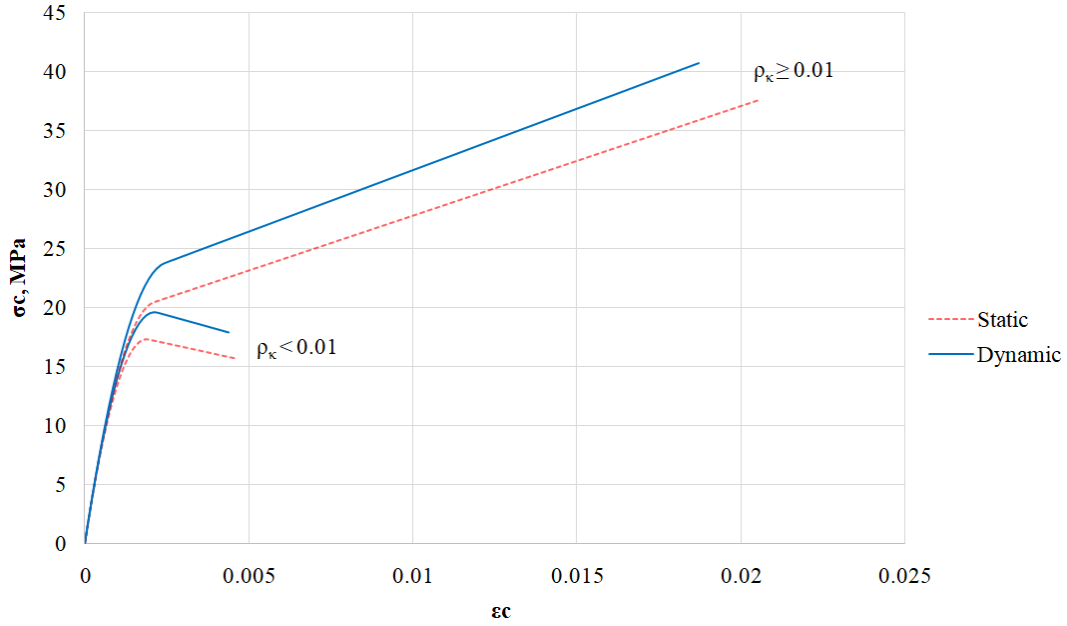
$$\varepsilon_{co}^d = k_{d,c}^\varepsilon f_{co} \quad (25)$$

$$R_f^d = k_{d,f}^R R_f \quad (26)$$

$$E_f^d = k_{d,f}^E E_f \quad (27)$$

The deformation of concrete confined by CFRP jacket follows the same pattern as in the case of static loading, and the deformation rate is taken into account by introducing four DIF

$$\begin{cases} k_{d,c}^f = 1 + (\lg \dot{\varepsilon} + 3) \cdot 0.03438 \\ k_{d,c}^\varepsilon = k_{d,c}^f \left[ 1 - 2 \cos \left( \frac{1}{3} \arccos \left( \frac{1}{(k_{d,c}^f)^2} - 1 \right) - \frac{2\pi}{3} \right) \right], & 10^{-3} \leq \dot{\varepsilon} \leq 10^2 \text{ s}^{-1} \\ k_{d,f}^R = (1 + 4.496 \cdot 10^{-4} \dot{\varepsilon}^{1.529}) \\ k_{d,f}^E = (1 + 5.634 \cdot 10^{-3} \dot{\varepsilon}^{0.8228}) \end{cases} \quad (28)$$



**Figure 1.** Diagrams of concrete confined by CFRP jacket at static and dynamic loading.

The remaining parameters are determined by the same formulas (2) - (8) as under static loading, in which the corresponding substitution of the index «d» is performed.

The above dependences are valid in the speed range from  $10^{-3}$  to  $10^2 \text{ s}^{-1}$ . At  $\dot{\varepsilon} = 10^{-3} \text{ s}^{-1}$  all DIF become equal to unity, which corresponds to the boundary of quasistatic loading. At speeds exceeding  $10^2 \text{ s}^{-1}$ , these dependences are not experimentally tested.

The diagram of the deformation of concrete confined by CFRP jacket at dynamic loading (at  $\rho_{\kappa}^d \geq 0.01$ ), which is described by the system (22), is presented in figure 1.

At the initial stage of loading, fairly close results were obtained, since, as already noted, the initial elastic modulus of concrete at static and dynamic compression were taken equal to each other. At this stage, the jacket is not yet stretched enough to significantly change the slope of the curved section of the diagram.

We note that in the place of the inflection of the dynamic diagram, not only the strength properties of limited concrete are higher, but also plastic deformations. This is caused by the specific properties of CFRP under the influence of dynamic load.

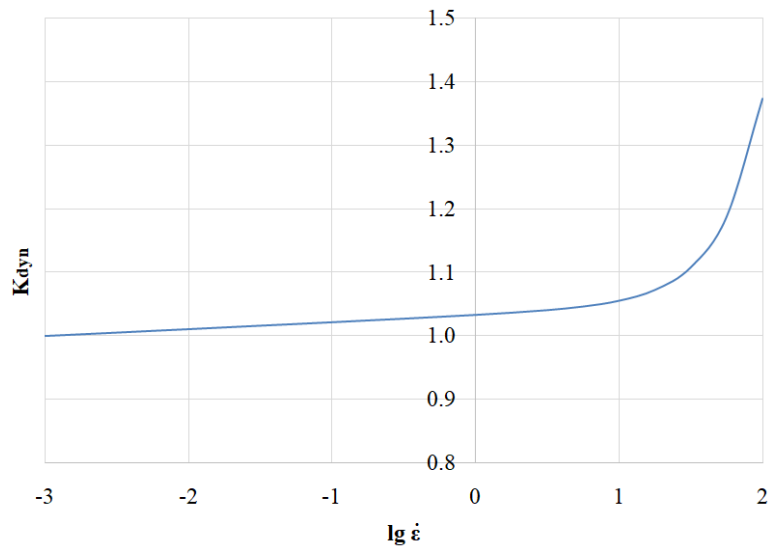
When moving to a straight section, the contribution to the work of the composite material increases. The slope of the linear section of the diagram is determined only by the increased mechanical characteristics of CFRP at dynamic loading. At strain rates of less than  $50 \text{ s}^{-1}$  the slope of the linear section does not differ significantly from that obtained under quasistatic loading, i.e.  $E_2^d \approx E_2$ .

The ultimate deformations of confined concrete at dynamic loading at the time of composite rupture are somewhat lower than with static.

In the case when  $\rho_{\kappa}^d < 0.01$  dynamic elastic modulus of the linear section of the diagram  $E_2^d$  is less than the corresponding static one  $E_2^d$ . The dynamic line approaches the static line with increasing strains.

We introduce the DIF for the ultimate strength of confined concrete

$$K_{dyn} = \frac{f_{cc}^d}{f_{cc}} \quad (29)$$



**Figure 2.** The dependence of DIF for the ultimate strength of confined concrete from the strain rate.

Analyzing the dependence presented in figure 2, we note that a significant increase in the bearing capacity of limited concrete is observed at strain rates of more than  $10 \text{ s}^{-1}$ .

The results obtained from the diagram require experimental confirmation.

Besides, these dependences are valid only for sufficiently short columns, when the influence of longitudinal bending at dynamic actions is not significant [25].

## 6. Conclusions

Following results were obtained:

1. Based on the assumption of the same law of deformation of concrete at static and dynamic loading, a deformation diagram of a concrete confined by CFRP jackets was obtained in the range of deformation rates of  $10^{-3} - 10^2 \text{ s}^{-1}$ . This diagram is a modification of the quasistatic diagram obtained in work [9].

2. Using the principle of the invariance of the potential energy of deformation of concrete at the time of its destruction at regime loading, a formula is obtained for determining DIF for ultimate relative strain of unconfined concrete  $k_{d,c}^{\varepsilon}$ .

3. It was revealed that a significant increase in the bearing capacity of confined concrete begins at strain rates of  $10 \text{ s}^{-1}$  or more.

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