Contribution of the Proper Orthogonal Decomposition Modes to Accuracy of Parameter Identification in Flexible Multibody Systems

Kensuke Hara, Nozomi Kitamura, Ryoya Nishi and Hiroshi Yamaura

May 19, 2024

Contribution of the proper orthogonal decomposition modes to accuracy of parameter identification in flexible multibody systems

Kensuke Hara*, **Nozomi Kitamura***, **Ryoya Nishi#**, **Hirhoshi Yamaura†**

* Department of Mechanical Engineering
Yokohama National University
79-5 Tokiwadai, Hodogaya-ku,
Yokohama, Kanagawa, 240-8501, Japan
hara-kensuke-zy@ynu.ac.jp

Sumitomo Heavy Industries
Material Handling Systems Co.,Ltd.
2-8-6 Nishishinbashi, Minato-ku,
Tokyo, 105-0003, Japan

†School of Engineering,
Tokyo Institute of Technology
2-12-1 Ookayama, Meguro-ku,
Tokyo, 152-8552, Japan
yamaura.h.aa@m.titech.ac.jp

Abstract

Acquisition of precise parameters is of key importance for accurate numerical simulation for multibody systems. However, it is generally difficult to obtain all parameters required for the simulation by only direct measurements. The authors have presented a parameter identification technique based on the adjoint method [1]. It incorporates the proper orthogonal decomposition (POD) into a cost function, in order to consider model uncertainties. More specifically, the proposed method uses data samples decomposed into the proper orthogonal decomposition modes (POMs) for the cost function.

This study considers a flexible beam supported by springs and dampers at its both ends as shown in Fig. 1. The springs and dampers are introduced as simple models for model uncertainties at the joint components such as friction, contact and so on. Table 1 indicates an example of the parameter identification test for the flexural rigidity EI (true value: 500 Pa) by the present method [1]. It can be found that the cost function given by relatively higher POMs leads precise estimated values, even though such higher POMs have quite lower contribution ratios. The objective of this study is to investigate the reason why the precise parameter estimation results could been obtained from the higher POMs.

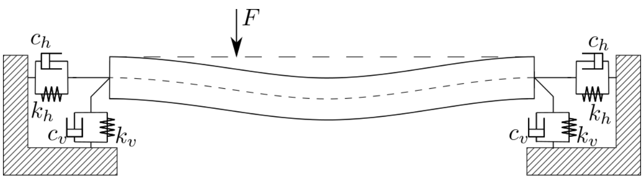

Figure 1: Flexible beam supported by springs and dampers

Table 1: Results of parameter identification for EI calculated by J_i^{POD} [1]

POM i (Contribution rate)	1 (9.96×10^{-1})	2 (2.05×10^{-3})	3 (1.77×10^{-3})	4 (3.81×10^{-4})	7 (1.40×10^{-5})	11 (9.46×10^{-7})
EI	–	–	502	500	500	500

We first outline the parameter identification method with the POD by the authors [1]. It introduces the cost function J defined as

$$J = \int_0^{T_\mathrm{s}} L\,dt, \qquad L = \frac{1}{2}[\boldsymbol{u}(t) - \bar{\boldsymbol{u}}(t)]^T[\boldsymbol{u}(t) - \bar{\boldsymbol{u}}(t)], \tag{1}$$

where $\bar{\boldsymbol{u}}$ denotes the measured data and $\boldsymbol{u}$ represents the system outputs. In addition, the present method applies the POD to a set of data samples $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}$. They can be expressed by the decomposition of the POMs as follows:

$$\bar{\boldsymbol{u}}(t) = \sum_{i=1}^{N_{\mathrm{d}}} \bar{a}_i(t)\bar{\boldsymbol{w}}_i, \qquad \boldsymbol{u}(t) = \sum_{i=1}^{N_{\mathrm{d}}} a_i(t)\boldsymbol{w}_i, \tag{2}$$

where $\bar{\boldsymbol{w}}_i$ and $\boldsymbol{w}_i$ denote the POMs obtained from $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}$, respectively. Since $\bar{a}_i$ and a_i can be calculated by $\bar{a}_i = \bar{\boldsymbol{u}}^T\bar{\boldsymbol{w}}_i$ and $a_i = \boldsymbol{u}^T\boldsymbol{w_i}$, the cost functions divided into each POMs can be defined as follows:

$$J_i^{\mathrm{POD}} = \int_0^{T_{\mathrm{s}}} L_i^{\mathrm{POD}}\, dt, \qquad L_i^{\mathrm{POD}} = \frac{1}{2}(a_i(t) - \bar{a}_i(t))^2. \tag{3}$$

For the model in Fig. 1, the equations of motion can be generally expressed by the differential algebraic equations (DAEs) as

$$\dot{\boldsymbol{q}} = \boldsymbol{v}, \quad [\mathbf{M}]\dot{\boldsymbol{v}} + \frac{\partial \boldsymbol{C}^T}{\partial \boldsymbol{q}}\boldsymbol{\lambda} = \boldsymbol{F}^{(e)}, \quad \boldsymbol{C}(\boldsymbol{q}) = \mathbf{0}. \tag{4}$$

The cost function divided into the ith POM in Eq. (3) is enhanced by incorporating the DAEs in Eq. (4) as follows[2]:

$$\hat{J} = J_i^{\mathrm{POD}} + \int_0^{T_{\mathrm{s}}} \boldsymbol{\xi}^T(\dot{\boldsymbol{q}} - \boldsymbol{v})dt + \int_0^{T_{\mathrm{s}}} \boldsymbol{\zeta}^T\left([\mathbf{M}]\dot{\boldsymbol{v}} + \frac{\partial \boldsymbol{C}^T}{\partial \boldsymbol{q}}\boldsymbol{\lambda} - \boldsymbol{F}^{(e)}\right)dt + \int_0^{T_{\mathrm{s}}} \boldsymbol{\mu}^T\boldsymbol{C}dt, \tag{5}$$

where $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$ and $\boldsymbol{\mu}$ represent the adjoint variables corresponding to the equations in Eq. (4). The unknown parameter $\boldsymbol{p}$ can be calculated by using the gradient $\nabla_{\boldsymbol{p}}\hat{J}$ obtained from the adjoint computation for the cost function (5).
This study investigates the relation between the POMs and the accuracy of the parameter identification. In order to derive the equations of motion, we employ the floating frame of reference formulation for the description the elastically supported beam shown in Fig. 1. Then, the time series data for the beam displacements are divided into the components of the POMs. Each components of POMs is analyzed by the Fourier analysis. After that, we compare the frequency components for the component of POMs with the analytical values of natural frequencies and discuss the contribution to the accuracy of the parameter identification.

Acknowledgements
This work is supported by JSPS KAKENHI Grant Number JP22K03979.

References

[1] Nishi, R.; Hara, K.; Yamaura, H.: Parameter identification method for multibody systems incorporating the adjoint method and the proper orthogonal decomposition. In ECCOMAS Thematic Conference of Multibody Dynamics, No. 242, Lisboa, Portugal, 2023.

[2] Nachbagauer, K.; Oberpeilsteiner, S.; Sherif, K.; Steiner, W.; The use of the adjoint method for solving typical optimization problems in multibody dynamics. J. Computational and Nonlinear Dynamics, 10-6: 061011, 2015.