A Generalised Signal Timing Scheme at Isolated Intersections Considering Multi-Modal Pedestrian Crossings

Guang Wang, Lijuan Wan, Chunhui Yu and Wanjing Ma

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G. Wang[a], L. Wan[b], C. Yu[a,*], and W. Ma[a]

[a] Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, Shanghai, China
wangguang518@tongji.edu.cn, hughyu90@tongji.edu.cn, mawanjing@tongji.edu.cn
[b] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong, China
lwanac@connect.ust.hk
[*] Corresponding author

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1 INTRODUCTION

Traffic congestion is a severe problem that has caused significant economic losses globally. For instance, in the United States, people wasted nearly 7 billion hours on commuting every year, resulting in an average cost of $160 billion (Schrank *et al.*, 2015). Traffic signals, serving as a backbone in large metropolitan cities, have shown a positive reception in controlling traffics and alleviating congestion. Real-time data based signal optimisations (e.g., actuated or adaptive signal control) are prevailing. However, their widespread implementation is hindered by poor data quality and burdensome maintenance (Wang *et al.*, 2024). In fact, over 95% of traffic signals in the United States and Canada still adopt the fixed-time signal control (Tang *et al.*, 2019).

Existing literature on signal optimisation generally allocates green durations to typical phase structures (e.g., NEMA dual-ring, eight-phase structure), with residual queues fully discharging (e.g., Yu *et al.*, 2017). Limited phase sequence options would not scale well under the unbalanced traffics. Meanwhile, only a limited number of studies have been historically dedicated to minimise a combination of vehicle and pedestrian delay. Majority rely on heuristic algorithms with estimating delays via micro-simulation tools such as VISSIM (e.g., Wang & Tian, 2010, Liang *et al.*, 2020). However, it is usually time-consuming and not suited for dynamic signal alteration. The elegant delay formulas and explicit models is needed.

This paper proposes a generalised signal timing scheme at an isolated intersection, which explicitly incorporates the benefits of vehicles and pedestrians in the objective function. Multi-modal pedestrian crossings is concerned, in which one-stage crossing (OSC) and two-stage crossing (TSC), together with shared and exclusive pedestrian phases appear in given platform. The general signal cycle may contain several sub-cycles. Low-demand phase could be skipped in sub-cycles and high-demand phase could have multiple right-of-way within the overall signal cycle. Over-saturated phases are allowed in sub-cycles. In this way, flexible phase sequence with sub-cycles, phase duration, and cycle length are optimised together to minimise the weighted average delay. The model is formulated as a mixed-integer non-linear programming (MINLP). To mitigate computational burden, we design an algorithm integrating Monte-Carlo tree search

(MCTS) and spatial branch-and-bound algorithm with outer approximation, where the former determines the phase sequence and the latter optimises the phase duration.

2 METHODOLOGY

Figure 1(a) depicts an isolated signalised intersection with four unique arms, including eight vehicle phases and six pedestrian phases (i.e., Q1-Q6). A candidate stage may contain two non-conflicting vehicle phases and the compatible pedestrian phases (e.g., s1-s8), or it may include all pedestrian phases without any vehicle phases (e.g., $s9$), as shown in Figure 1(b). Given N time-slots in a general signal cycle (e.g., $N = 7$ in Figure 1(c)), the stage and duration for each time-slot are the decision variables to be optimised. In this case, the phase schemes for both vehicle and pedestrian flows are naturally formed if their corresponding minimum green duration constraints are satisfied, as illustrated in Figure 1(d). Note that a stage can appear repetitively and even consecutively in a cycle. Repetitiveness guarantees phase skipping for low-demand phases and multiple right-of-way for high-demand phases. Consecutiveness guarantees model compatibility (i.e., the feasible area with a smaller number (i.e., $\leq N$) of time-slots is a subset of that from with N time-slots). With no limit on the number of time-slots, the phase structure becomes more flexible and can accommodate arbitrary traffics.

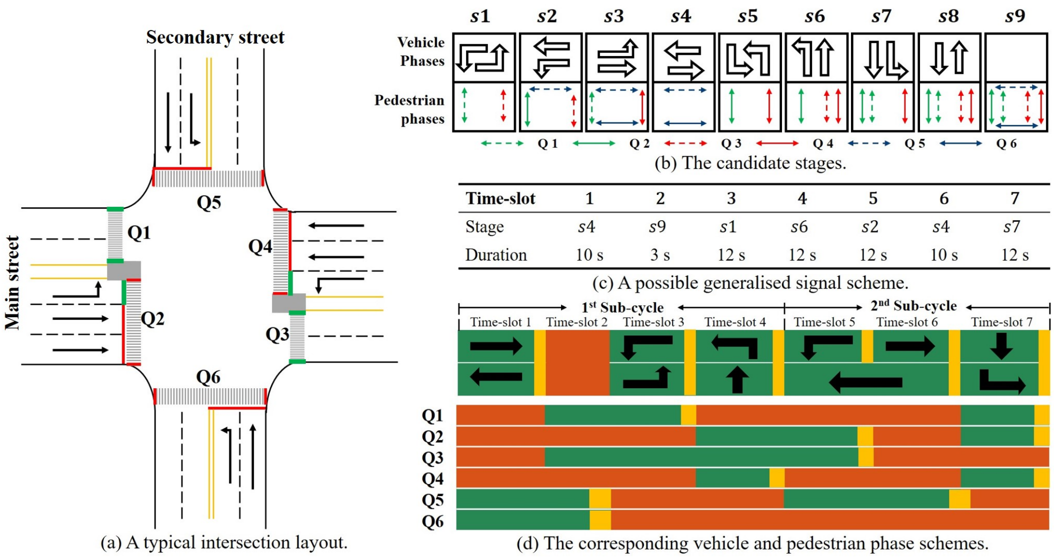


Figure 1 – *The framework of generalised signal timing scheme.*

From the viewpoint of vehicle phases, Figure 1(d) depicts two sub-cycles. Phases in secondary street are skipped and that in main street have multiple right-of way. The latter one may allow to have over-saturated phases (i.e., having residual queues). However, each vehicle movement are deemed to be under-saturated within the general cycle. From the viewpoint of pedestrian phases, taking Q1 as an example, pedestrians could cross the street using shared phases (i.e., time-slots 3 and 7) and exclusive phase (i.e., time-slots 2). Meanwhile, the green duration could merge if two adjacent time-slots guarantee the right-of-way for the same phase.

Based on point-queue model, we could deduce the delay formulas of vehicle and pedestrian OSC in a straightforward way. Whereas, the flexible phase scheme for pedestrians results in various TSC delay expressions. Taking Q1 as an example, denote $[\widetilde{tg}_1^n + \tau_1, \widetilde{tr}_1^n]$ as the time interval for pedestrians arriving at centra refuge island. $\widetilde{tg}_2^n$ and $\widetilde{Ntg}_2^n$ are the start times of green and next green phases for the second-stage crossing. $\widetilde{tr}_2^m - \tau_2$ is the start time of Green Flashing, where pedestrians stop crossing. The delays at refuge island can break into three categories: (i) $\widetilde{tr}_2^m - \tau_2 \geq \widetilde{tr}_1^n$ or $\widetilde{Ntg}_2^m \leq \widetilde{tg}_1^n + \tau_1$, where delay equals to zero. (ii) $\widetilde{tr}_2^m - \tau_2 \leq \widetilde{tg}_1^n + \tau_1$ and

$\widetilde{Ntg}_2^m \geq \tilde{tg}_1^n + \tau_1$ (i.e., the shade area in Figures 2(a) and (b)). (iii) $\tilde{tg}_1^n + \tau_1 \leq \tilde{tr}_2^m - \tau_2 \leq \tilde{tr}_1^n$ and $\widetilde{Ntg}_2^m \geq \tilde{tg}_1^n + \tau_1$ (i.e., the shade area in Figures 2(c) and (d)). The overall objective function to be minimised is defined as $\rho \sum_{p \in P} \frac{D_p^{veh}}{a_p^{veh} C} + (1-\rho) \sum_{q \in Q} \frac{D_q^{ped}}{a_q^{ped} C}$, where D_p^{veh} and D_q^{ped} are delays of vehicle and pedestrian respectively. a_p^{veh} and a_q^{ped} are corresponding traffic demands. C is the signal cycle length.

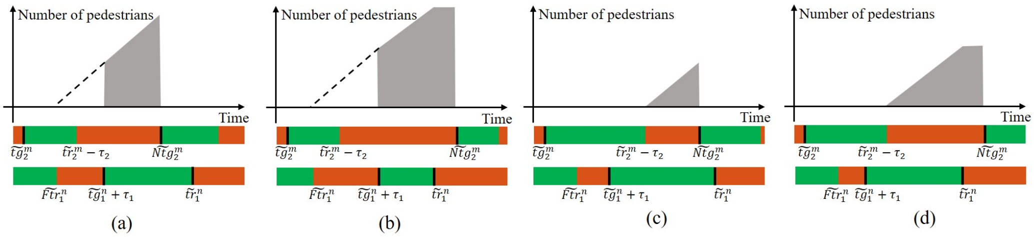


Figure 2 – *Sketch of delays for pedestrians arriving at the refuge island.*

The formulated MINLP model can be directly solved by optimization solvers (e.g., Gurobi), but it suffers from the curse of dimensionality especially with a large number of time-slots. Therefore, a hybrid algorithm is designed to find a near-optimal solution. Given the superiority of MCTS in solving Markov decision problems (Coulom, 2007), it is adopted to determine the stage sequence. In the first step (Selection), a most expandable node is chosen considering the trade-off between exploitation and exploration. With a stage sequence generated in the step of Simulation, Gurobi is used to optimise the green duration for each time-slot. The minimised weighted vehicle and pedestrian delay is propagated as an evaluation index to update the node values. By iterating the four steps, a near-optimal stage sequence, as well as the optimal green durations and cycle length, can be obtained.

3 RESULTS

Figure 3 shows the primary results about the proposed model and algorithm. The findings are:

(1) The proposed solution algorithm could achieve a balanced trade-off between solution quality and computational burden (see Figure 3(a)). It could provide a near-optimal signal scheme within ~100 s. Compared with MCTS, it guarantees the solution robustness.

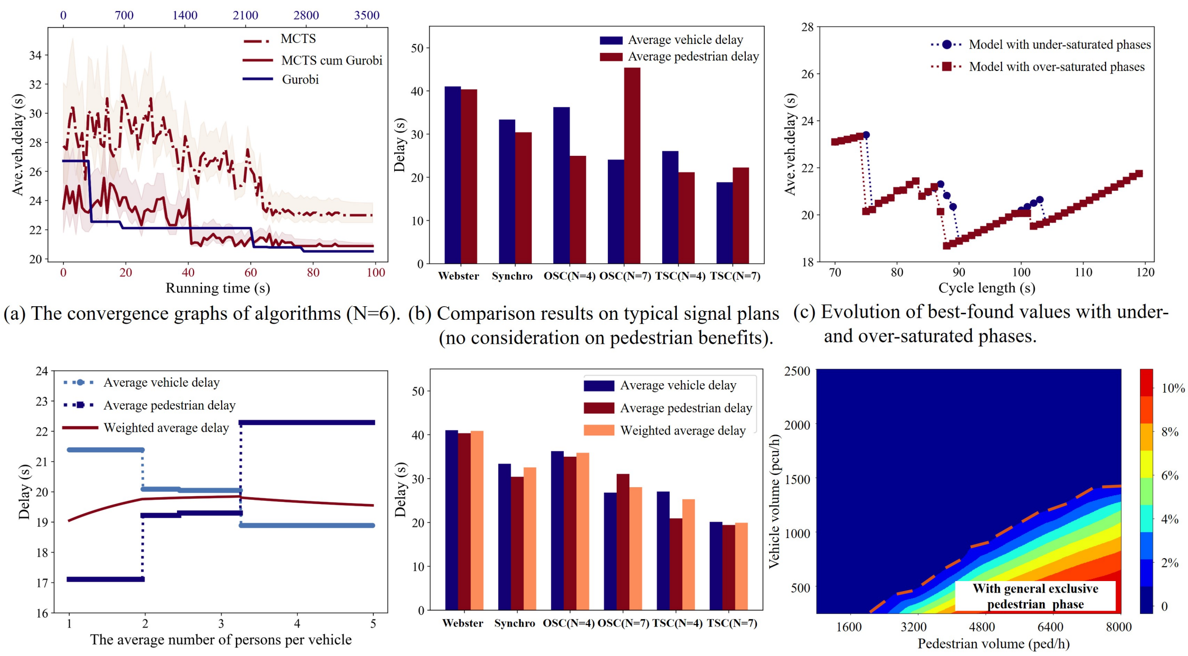
(2) Figure 3(b) demonstrates the results of vehicle-oriented optimisation. In line with our expectation, the models with $N = 7$ are always better than that with $N = 4$, having lower average vehicle delay whether in OSC or TSC. Compared with Webster and Synchro plans, TSC with $N = 7$ can both reduce vehicle and pedestrian delay by 53% and 45%, 43% and 27%, respectively. Note that the results incorporating pedestrian benefits are shown in Figures 3(d)-(e).

(3) Figure 3(c) shows the detailed average vehicle delay from models with and without allowing over-saturation in sub-cycles. The proposed model yields a superior overall optimal result.

(4) The weight ρ is tacitly chosen based on individuals, i.e, $\sum_{q \in Q} a_q^{ped} / (\mu \sum_{p \in P} a_p^{veh} + \sum_{q \in Q} a_q^{ped})$, where μ is the average number of persons per vehicle. It is observed from Figure 3(e) that the signal schemes are relatively robust within an acceptable variance on μ.

(5) When incorporating the pedestrian benefit with $\mu = 3$, the proposed model could achieve a balanced performance in terms of both vehicles and pedestrians (see Figure 3(e)). Particularly, TSC with $N = 7$ could significantly improve the system service of intersection, with 51%, 39%, 44%, 29%, and 21% reduction on delays per person.

(6) Figure 3(e) presents a comparative scenario with and without the general exclusive pedestrian phase (i.e., s9 in Figure 1) in the modelling process. It suggests that setting exclusive phase is necessary under lower vehicle streams and higher pedestrian streams on the main street.



(a) The convergence graphs of algorithms (N=6). (b) Comparison results on typical signal plans (no consideration on pedestrian benefits). (c) Evolution of best-found values with under- and over-saturated phases.

(d) Delay changes of TSC under different weights. (e) Comparison results under the premise of 3 persons per vehicle. (f) The application domain of general pedestrian exclusive phases.

Figure 3 – *The relative results of numerical experiments.*

4 DISCUSSION

From the modelling perspective, the results show that the flexible phase structure with sub-cycles assists to maximise the green utilisation. Setting over-saturated phases provides an extended feasible area where further delays reduction can be observed. The application domain of exclusive phase modelling provides operational guidelines for intersection manager. From the computation perspective, the iterative regeneration for phase sequence and duration in this research is a promising decision support tool to signal optimisation problem. Future research can be implemented on the time-variant traffics by incorporating stochastic arriving pattern. Trajectory based data could be utilised to calibrate the spatial-temporal delay modelling.

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