Preservice Teachers in Real-World Problem-Posing: Can They Turn a Context into Mathematical Modelling Problems?

Ahmad Wachidul Kohar, Dayat Hidayat, Nina Rinda Prihartiwi and Evangelista Lus Windyana Palupi

August 30, 2022

Preservice Teachers in Real-world Problem-Posing: Can They Turn a Context into Mathematical Modelling Problems?

Ahmad W. Kohar[1,*] Dayat Hidayat[1] Nina R. Prihartiwi[1] Evangelista L.W. Palupi[1]

[1] *Universitas Negeri Surabaya*
[*]*Corresponding author. Email: ahmadkohar@unesa.ac.id*

ABSTRACT

While problem-posing respecting real-world situations can be a promising approach for fostering modelling competence, research on modelling through problem posing is scant. This present paper aims to characterize the mathematical tasks designed by prospective teachers regarding the criteria of a modelling problem. Data were collected from fifty mathematical tasks posed by twenty-five preservice teachers as participants at a public university in Surabaya, Indonesia, within a summative test of an assessment course. The problem-posing task asked the participants to pose two different mathematical tasks from a given real-world situation. To analyze, the participants' responses were coded as solvable or unsolvable tasks and then further coded regarding two aspects of modelling problem i.e., connection to reality and openness of a problem. Our analysis revealed that the participants tended to pose problems with authentic connections rather than artificial connections to reality. However, only a few of the posed problems were indicated to promote openness in terms of either various mathematical models or an unclear initial state, which is the crucial indicator of a modelling problem. Implications regarding modelling competence via problem-posing in preservice teacher education are discussed.

Keywords: *Mathematical modelling, Preservice teacher, Problem-posing*

1. INTRODUCTION

In many countries, mathematical modelling, which is regarded as the process of making sense of a real-world situation mathematically and using several iterations of problem-solving to find a solution (Lesh & Doerr, 2003), is currently posited as a crucial part of all levels of mathematics education (Krawitz et al, 2022; Yang et al, 2022). It is now more crucial than ever for teacher preparation programs to make sure that they are equipping preservice teachers with the skills necessary to effectively encourage fruitful mathematical modelling in their classrooms. According to Paolucci and Wessels (2017), teacher preparation programs should provide interventionist programs that not only could encourage preservice teachers' metacognitive recognition of the thinking processes and behaviors that characterize mathematical modelling, but also possess to design tasks and problems that successfully include their future students in the process of mathematical modelling.

Designing a mathematical modelling task is known as a challenging activity since some criteria need to be fulfilled by the designer when posing such kind of task. Various criteria for modelling problems were proposed by some scholars. Lesh, Hoover, et al (2006)'s framework, one of the most often cited ones, presents six guidelines for structuring a modelling task. These include the simple-prototype principle, the reality principle, the model-construction principle, the self-evaluation principle, the model-documentation principle, and the model-generalization principle. The reality principle is also highlighted by Hartmann et al (2021) when assessing students' created mathematical modelling tasks. Within their analytical framework, the aspect of problem openness, which is also concerned by Maaß (2010) and Yeo (2017), becomes another crucial aspect of the evaluation of the extent to which a posed mathematical task can be coded as a modelling task.

Research on assessing modelling tasks has given special attention to the criteria of connection to reality and openness in recent years. Stohlmann et al (2017) stressed that the aspect of reality indicates the potential of modelling tasks which can promote students to interpret the activity meaningfully given their varying degrees of mathematical proficiency and general knowledge. It is also noted that this aspect reflects realistic scenarios that make sense in terms of students' real-life experiences and knowledge (Hartman et al, 2021; Unver et al, 2018; Turner et al, 2022). This is in line with the conceptualization of reality connection revealed by Palm (2007) highlighting that when a problem asks a question that is crucial to the real-world scenario being portrayed and thus accurately reflects significant elements of that circumstance, it has an authentic connection to reality. When a problem does not address essential elements of the situation in the actual world, it is regarded to have an artificial connection, and if it is purely mathematical in nature, it is deemed to have no link to reality. Meanwhile, the openness of a problem indicates the use of various mathematical models or an unsettled starting point (Silver, 1995). From a wider perspective, the openness of tasks shows the possibility of various goals, methods, task complexities, answers, and extensions, as well as how different types of tasks may affect student learning (Yeo, 2017). In this paper, we considered the openness of problems related to two main aspects, namely whether the mathematical models might be constructed by solvers are various or not and whether the initial state was clear or not as an unclear initial state is a feature of modelling challenges (Maaß 2010). The clarity of initial state is also closely related to knowledge about the real world with extraneous or missing data, in which a solver of modelling problems should rely on assumptions when there is a lack of knowledge or given information (Hartmann et al, 2021).

In conjunction with the criteria of modelling problem, preservice teachers meet challenges to create a such problems. To meet these challenges, scholars suggest problem-posing as activities that guides preservice teachers to create their modelling problems for school learning. For preservice teachers, posing modelling problem is not only intended to improve their modelling competences, which in this regard, Blum and Niss (1991) argued that problem-posing may be helpful for modelling since it can be viewed as a problem-solving activity, but also to improve their skills in designing problem for being implemented within their future teaching practices. While problem posing based on real-world events might be a promising strategy for encouraging students' modelling competence due to its strong connection to problem-solving (Chen et al. 2013; Hartmann et al, 2021), there is still few research on encouraging students' modelling through problem posing. Additionally, research on exploring preservice teachers' performance in posing modelling problems are still lacking although some are reported to show current trend of modelling task design in recent years (See e.g., Geiger et al, 2022; Paredes et al, 2020; Paolucci et al, 2017; Turner et al, 2022).

Research found that preservice teachers frequently struggle with posing mathematical tasks such as being limited to pose simple, one-step tasks, exercises, tasks with lower-level cognitive demands (procedures without connections), tasks with a narrow domain, even non-mathematical or unsolvable tasks, which included bare or insufficient information (Fitriana et al., 2022; Kohar et al., 2022). In addition, when confronted with real-world situations, preservice teachers often find difficulties in turning a context into modelling problems (Paolucci, 2017; Ortiz & Ferri, 2022). In conclusion, problems with task solvability, task fostering various mathematical models, and task connection to reality become critical for potential teachers' encounters with problem posing.

Problem-posing is defined as "both the formulation of new problems and the re-formulation of existing ones" (Silver, 1994). One can categorize whether problem-posing occurs prior to (pre-solution), during (within-solution), or following (post-solution) problem solving (Silver, 1994). Another distinction between problem posing in free situations (for example, posing a problem with no restrictions), semi-structured situations (for example, posing a problem based on a given situation), and structured problem-posing situations (for example, reformulating the given problem) can be created (Stoyanova, 2000). The present study addresses this research gap by examining preservice teachers' problem-posing performance based on a given situation in a real-world setting which is potentially constructed as the basis of posing modelling problem. More specifically, the aim of this study is to describe the types of mathematical problems pre-service teachers pose in each real-world situation.

2. RESEARCH METHOD

2.1. Context and Participants

Explorative research was chosen as the research methodology to investigate preservice teachers' product of problem-posing, namely mathematical tasks. As many as twenty-five preservice teachers (7 males, 18 females) who have completed more than half of the total course credits at an undergraduate program in mathematics education at a public university in Surabaya Indonesia, were involved in an interventionist program for developing their knowledge and experiences in math assessment. Thus, they have gained some basic skills in designing mathematical tasks, such as task format and cognitive processes in task design as the basis for designing any mathematical problems. However, they did not join any explicit courses on how to design modelling problems.

2.2. Instruments and Data Collection

The instrument consists of a problem-posing task where participants were requested to employ a context of online taxi driver salary to pose two different mathematical tasks. The task was completed by the participants in the summative test of the assessment course. An assessment expert from the same university as the first author was asked to analyze the instrument's initial draft and provide qualitative feedback on the areas of content, construct, and language concerns to verify the task. Figure 1 shows the problem-posing task. The context of the problem in the task within Figure 1 is well recognized by preservice teachers.

2.3. Data Analysis

The frequency of tasks posed by the preservice teachers was categorized according to two domains: connection to reality and problem openness. Each created problem was first evaluated before being classified into those two domains. Unsolvable problems (tasks) include confusing language, unexplained crucial premises, insufficient contextual, numerical, figural, tabular, or pictorial information concerning the provided context (online taxi driver salary) that causes it impossible to solve was also considered to be unsolvable (Kohar et al, 2022).

To examine the reliability of the coding, the authors applied Cohen's Kappa for each of the codings from the coders, which are the first, the second, and the third author. Cohen's Kappa (κ) resulted from analyzing the coding statistically. The resulted κ score is respectively 0.874 (n=44), 0.867 (n=44), and 0.642 (n=50) for the solvability of the task, connection to reality, and openness of the problem. According to Landis and Koch (1997), this result indicates that the coding is almost perfect for both domains of solvability of task and connection to reality the two raters, while that is substantial for the domain of openness of the problem. Hence, the authors negotiated the coding primarily on the domain of the level of cognitive processes to increase the agreement. To report, the codings of the

ONLINE TAXI DRIVER SALARY

A digital transportation company implements three salary schemes for its drivers, namely main income, points, and additional commissions. The following table shows information regarding the payroll scheme.

Main Income

Distance traveled	Income
0-10 km	IDR 10.000
>10 km	IDR 2.000 per km

Points

Types	Points	Note
Basic points	+1	Obtained from every taking orders from customers at any time
Bonus points	+2	Obtained for orders during peak hours (06.00-09.00 and 16.00-20.00)

The maximum number of points per order is 3 points

Daily Bonus Points Exchange

12-14 points = IDR10,000	21-27 points = IDR50,000
15-16 points = IDR30,000	28-30 points = IDR70,000
17-20 points = IDR40,000	

The maximum points that can be exchanged per day are 30 points.

Additional commissions

Additional IDR 3,000 for every order at 23.00-05.00

Pose two different mathematical problems using the information above. Also, write down alternative answers for each of these problems to help you check the solvability of the problems.

Figure 1. Problem-posing task

second author were then selected to be further analyzed and reported.

promote various asssumptions or various models or strategies.

Table 1. Analytical Framework of Coding Schemes

Solvability	Connection to reality	Openness
➢ Solvable (Problems can be solved by mathematics, no matter the sentence structure is easy to understand or not, whether the problem is contextual [or not]), ➢ Unsolvable (Cannot be solved due to lack of information, incorrect mathematical assumptions, or everyday contexts that make it unsolvable)	➢ Authentic (occurs in the real world), ➢ Artificial (not really happening in the real world, camouflage context, without context the problem can still be solved)	➢ Closed Problem (Single Answer and Method) ➢ Open problem: more than one mathematical models (at least two different mathematical models) ➢ Open problem: various initial state (there are assumptions required, information needed but not known) ➢ Open problem: more than mathematical models & various initial states (satisfies both)

3. RESULTS

3.1. Distribution of Problems

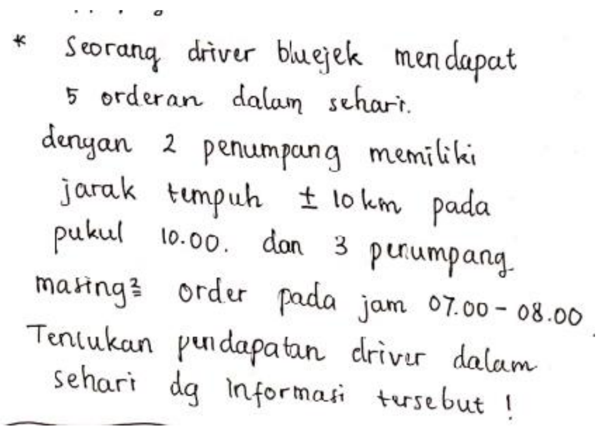
Of the fifty responses, there were 44 items (88.0%) that are solvable, while the remaining 6 items (9.09%) are unsolvable. Since we were only interested in the solvable tasks, we did not consider the unsolvable tasks for further analysis. Table 2 indicates the distribution of solvable tasks regarding the level of context use and level of cognitive processes.

Table 2. Distribution of problem

Types	N	Percentage (%)
Solvable	44/50	88.00%
Artificial Connection	4/44	9.09%
Authentic Connection	40/44	90.91%
Closed Problem	35/44	79.55%
Open problem: more than one mathematical models	3/44	6.82%
Open problem: various initial state	5/44	11.36%
Open problem: more than mathematical models & various initial state	1/44	2.27%

Table 2 shows that most of the problems posed by the preservice teachers are regarded as solvable (88%), while only four of them are coded as problem with artificial connection. Meanwhile, more than 90% of them included authentic connection. With regard to the openness of problem, most of the posed problems have only one mathematical model or strategy to solve, while only 9 of them contains information which

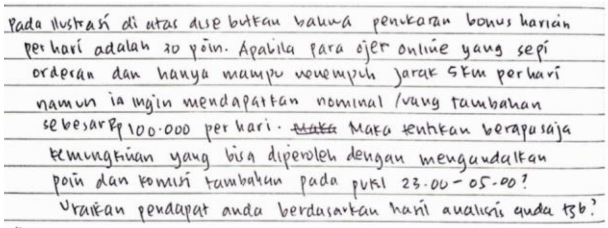
3.2. Unsolvable Problem



Translation: A driver gets orders for ojek five times a day, namely two passengers with 10 km at 10:00 and three passengers each at 07:00-08:00. Determine the driver's income in a day.

Figure 2. Unsolvable problem

Problem in Figure 2 is unsolvable since there is missing information about the distance travelled by the driver regarding the contextual information about taking three passengers who ordered online taxi driver. Thus, the problem is unsolvable.

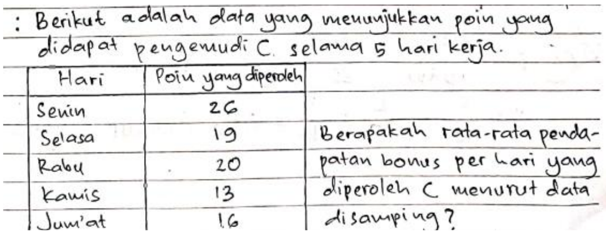


Translation: In the illustration, it is known that the maximum daily point exchange is 30 points. If an online motorcycle taxi driver is low on orders that day, and is only able to cover a distance of 5 km, while he wants to earn IDR100,000 by relying on additional points and commissions at 23.00-05.00, determine how likely the income he will get? Explain your strategies.

Figure 3. Unsolvable Problem

Another example of unsolvable problem is given by Figure 3, showing that it is mathematically unsolvable due to the missing interpretation of the use of bonus points that a driver might get when the distance traveled is only 5 km, which is impossible to get additional points for exchange.

3.3. Artificial Connection


: Berikut adalah data yang menunjukkan poin yang didapat pengemudi C selama 5 hari kerja.

Hari	Poin yang diperoleh
Senin	26
Selasa	19
Rabu	20
Kamis	13
Jum'at	16

Berapakah rata-rata pendapatan bonus per hari yang diperoleh C menurut data disamping?

Translation: The following table indicates the total points got by driver C in five weekdays.

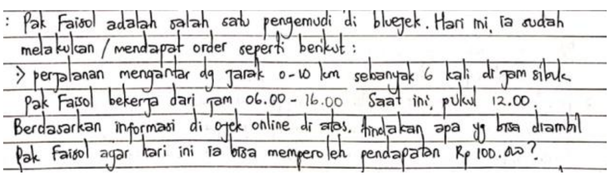
Day	**Points**
Monday	26
Tuesday	19
Wednesday	20
Thursday	13
Friday	16

What is the average bonus points obtained by driver C ?

Figure 4. Artificial connection Problem

Whether or not the situation of getting points from driving activity in the online taxi driver company, the performance of a solver of problem in Figure 4 will not be affected by such a situation. It means, the context of the problem is artificially wrapped in the problem.

3.4. Authentic Connection


: Pak Faisol adalah salah satu pengemudi di bluejek. Hari ini, ia sudah melakukan / mendapat order seperti berikut :
> perjalanan mengantar dg jarak 0-10 km sebanyak 6 kali di jam sibuk.
Pak Faisol bekerja dari jam 06.00 - 16.00 Saat ini, pukul 12.00.
Berdasarkan informasi di ojek online di atas, tindakan apa yg bisa diambil Pak Faisol agar hari ini ia bisa memperoleh pendapatan Rp 100.000?

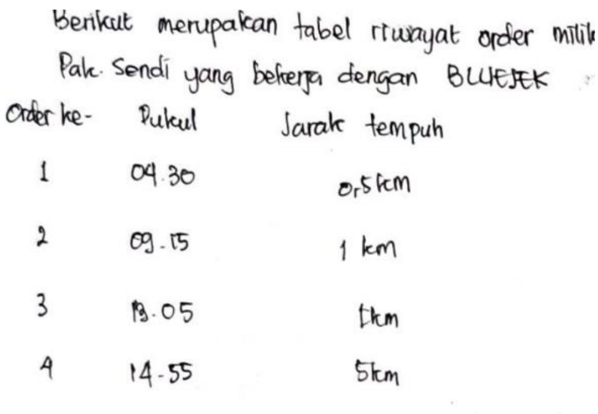
Mr Faisal is an online motorcycle taxi driver. Today, he received an order with the following details.

The trip takes passengers with a distance of 0-10 km 6 times during peak hours until 12.00. He works from 06.00-16.00. If he wants to earn IDR100,000 that day, what action can he take?

Figure 5. Authentic connection problem

Figure 5 indicates a problem with authentic connection since the context of number of work hours, constraints of targeted income, and number of orders are used to solve the problem of finding possibilities of schemes to get the targeted income of IDR100,000.

3.4. Closed Problem


Berikut merupakan tabel riwayat order milik Pak Sendi yang bekerja dengan BLUEJEK

Order ke-	Pukul	Jarak tempuh
1	04.30	0,5 km
2	09.15	1 km
3	13.05	1 km
4	14.55	5 km

Translation: he following indicates the number of orders by Pak Sandi who works as a driver in the online ojek company.

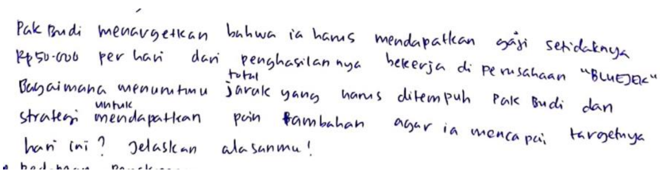
Order 1	04.30	0.5 km
Order 2	09.15	1 km
Order 3	13.05	1 km
Order 4	14.55	5 km

Find the total income that Pak Sandi got in that day.

Figure 6. Closed problem

The problem in figure 6 is the most typical problem found in the participants' responses. Although it is simple problem, the interpretation of the information given in the table of illustrations is rather complicated so that the solver of this problem can implement the rules of giving salary to the driver correctly.

3.5. Open Problems


Pak Budi menargetkan bahwa ia harus mendapatkan gaji setidaknya Rp50.000 per hari dari penghasilannya bekerja di Perusahaan "BLUEJEK". Bagaimana menurutmu total jarak yang harus ditempuh Pak Budi dan strategi untuk mendapatkan poin tambahan agar ia mencapai targetnya hari ini? Jelaskan alasanmu!

Translation: Pak Budi targets that he should earn at least IDR 50,000 per day from his income as a driver at an online motorcycle taxi company. In your opinion, what is the total distance and additional bonus points he must get to meet the target? Determine the strategy he needs to formulate and implement to achieve his target.

Figure 7. Open Problem

The problem in Figure 7 is an example of a problem that is closely related to the criteria of the modeling problem. Not only about its strong connection to reality, but also its chances to open the solver of this problem to create more than one mathematical model or strategy to find the targeted income. Also, various assumptions can be generated to support the reasoning of getting the solution of this problem.

5. DISCUSSION AND CONCLUSION

The study's findings show that it can be difficult to pose problems using online taxi driver salary. This is demonstrated by the data, which indicate that there weren't many problems that fit the criteria for openness, either because they're connected to various initial assumptions or mathematical models/strategies. The following is a discussion of several potential problems with this conclusion and the potential impact of the context.

The results of this study serve as a crucial wake-up call for teacher education, urging future educators to develop their capacity for posing mathematical challenges that satisfy the requirements of modelling tasks. Problem posing is a multifaceted and challenging task (Crespo & Sinclair, 2008), so problems with this phenomenon should be addressed by offering interventions that can enhance aspiring teachers' capacity of turning contexts into authentic modelling problem, decrease their propensity to use excessive amounts of information provided in the tasks, unfamiliar terms, unspecific units of contexts, and unacceptable use of mathematical symbols (Zulkardi & Kohar, 2018), and collaborative problem-posing (Crespo, 2020; Utami & Hwang, 2021).

To conclude, the findings of this research show that the preservice teachers tended to pose problems with authentic connections rather than artificial connections to reality. However, only a few of the posed problems were indicated to promote openness in terms of either various mathematical models or an unclear initial state, which is the crucial indicator of a modelling problem.

ACKNOWLEDGMENTS

We acknowledge the contribution from all the participants of this study.

REFERENCES

Crespo, S. (2020). Learning to pose collaborative mathematics problems with secondary prospective teachers. *International Journal of Educational Research*, *102*, 101430. https://doi.org/10.1016/j.ijer.2019.05.003

Geiger, V., Galbraith, P., Niss, M., & Delzoppo, C. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies. *Educational Studies in Mathematics*, *109*(2), 313-336.

Kohar, A. W., Rahaju, E. B., & Rohim, A. (2022). Prospective teachers' design of numeracy tasks using a physical distancing context. *Journal on Mathematics Education*, *13*(2), 191-210.

Krawitz, J., Chang, Y. P., Yang, K. L., & Schukajlow, S. (2022). The role of reading comprehension in mathematical modelling: improving the construction of a real-world model and interest in Germany and Taiwan. *Educational Studies in Mathematics*, *109*(2), 337-359.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *biometrics*, 159-174. https://doi.org/10.2307/2529310

Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Routledge.

Maaß, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik*, *31*(2), 285-311.

Ortiz, C. G., & Ferri, R. B. (2022). Pre-service teachers' challenges in implementing mathematical modelling: Insights into reality. *PNA. Revista de Investigación en Didáctica de la Matemática*, *16*(4), 309-341.

Palm, T. (2007). Features and impact of the authenticity of applied mathematical school tasks. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education. The 14th ICMI Study* (vol.

10, pp. 201–208). Boston, MA: Springer US. https://doi.org/10.1007/978-0-387-29822-1_20

Paredes, S., Cáceres, M. J., Diego-Mantecón, J. M., Blanco, T. F., & Chamoso, J. M. (2020). Creating Realistic Mathematics Tasks Involving Authenticity, Cognitive Domains, and Openness Characteristics: A Study with Pre-Service Teachers. *Sustainability*, *12*(22), 9656.

Paolucci, C., & Wessels, H. (2017). An Examination of preservice teachers' capacity to create mathematical modeling problems for children. *Journal of Teacher Education*, *68*(3), 330-344.

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics*, *14*(1), 19-28.

Silver, E. A. (1995). The Nature and Use of Open Problems in Mathematics Education: Mathematical and Pedagogical Perspectives. *Zentralblatt fur Didaktik der Mathematik/International Reviews on Mathematical Education*, *27*(2), 67-72.

Stohlmann, M., Maiorca, C., & Allen, C. (2017). A Case Study of Teachers' Development of Well-Structured Mathematical Modelling Activities. *Mathematics Teacher Education and Development*, *19*(2), 4-24.

Stoyanova, E. (2000). Empowering students' problem solving via problem posing: the art of framing'good'questions. *Australian Mathematics Teacher*, *56*(1), 33-37.

Turner, E. E., Bennett, A. B., Granillo, M., Ponnuru, N., Roth Mcduffie, A., Foote, M. Q., ... & McVicar, E. (2022). Authenticity of elementary teacher designed and implemented mathematical modeling tasks. *Mathematical Thinking and Learning*, 1-24.

Unver, S. K., Hidiroglu, C. N., Dede, A. T., & Guzel, E. B. (2018). Factors revealed while posing mathematical modelling problems by mathematics student teachers. *European Journal of Educational Research*, *7*(4), 941-952.

Utami, I. Q., & Hwang, W. Y. (2021). The impact of collaborative problem posing and solving with ubiquitous-decimal app in authentic contexts on math learning. *Journal of Computers in Education*, 1-28. https://doi.org/10.1007/s40692-021-00209-5

Yang, X., Schwarz, B., & Leung, I. K. (2022). Pre-service mathematics teachers' professional modeling competencies: a comparative study between Germany, Mainland China, and Hong Kong. *Educational Studies in Mathematics*, *109*(2), 409-429.

Yeo, J. B. (2017). Development of a framework to characterise the openness of mathematical tasks. *International Journal of Science and Mathematics Education*, *15*(1), 175-191.

Zulkardi, Z., & Kohar, A. W. (2018). Designing PISA-like mathematics tasks in Indonesia: Experiences and challenges. In *Journal of Physics: Conference Series* (Vol. 947, No. 1, p. 012015). IOP Publishing.

[9] A. Pnueli, In transition from global to modular temporal reasoning about programs, in: K.R. Apt (Ed.), Logics and Models of Concurrent Systems, Springer, Berlin, Heidelberg, 1984, pp. 123–144. DOI: https://doi.org/10.1007/978-3-642-82453-1_5

[10] B. Meyer, Applying "Design by Contract", Computer 25(10) (1992) 40–51. DOI: https://doi.org/10.1109/2.161279

[11] S. Bensalem, M. Bogza, A. Legay, T.H. Nguyen, J. Sifakis, R. Yan, Incremental component-based construction and verification using invariants, in: Proceedings of the Conference on Formal Methods in Computer Aided Design (FMCAD), IEEE Press, Piscataway, NJ, 2010, pp. 257–256.

[12] H. Barringer, C.S. Pasareanu, D. Giannakopolou, Proof rules for automated compositional verification through learning, in Proc. of the 2nd International Workshop on Specification and Verification of Component Based Systems, 2003.

[13] M.G. Bobaru, C.S. Pasareanu, D. Giannakopoulou, Automated assume-guarantee reasoning by abstraction refinement, in: A. Gupta, S. Malik (Eds.), Proceedings of the Computer Aided Verification, Springer, Berlin, Heidelberg, 2008, pp. 135–148. DOI: https://doi.org/10.1007/978-3-540-70545-1_14